Section 6.9: Hyperbolic Functions

Definition: Hyperbolic Trig Functions (see page 477 for more):

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Identities: (see page 478 for more)

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

Derivative Formulas for Hyperbolic Functions: (see pages 478-479 for more)

$$\frac{d}{dx}[\cosh u] = (\sinh u) \cdot u'$$

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$$\frac{d}{dx}[\tanh u] = \left(\operatorname{sech}^2 u\right) \cdot u'$$

$$\frac{d}{dx}[\operatorname{sech} u] = (\operatorname{sech} u \tanh u) \cdot u'$$

Integral Formulas for Hyperbolic Functions: (see page 480 for more)

$$\int \sinh u \, du = \cosh u + C$$

$$\int \cosh u \, du = \sinh u + C$$

$$\int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$\int \operatorname{sech} u \tanh u \, du = \operatorname{sech} u + C$$

Derivative Formulas for Inverse Hyperbolic Functions: (see pages 481-482 for more)

$$\frac{d}{dx}\left[\sinh^{-1} x\right] = \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx}\left[\cosh^{-1} x\right] = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\left[\tanh^{-1} x\right] = \frac{1}{1 - x^2}$$

Integrals Involving Inverse Hyperbolic Functions:

$$\int \frac{1}{\sqrt{u^2+1}}\,du = \sinh^{-1} u + C$$

$$\int \frac{1}{\sqrt{u^2-1}}\,du = \cosh^{-1} u + C$$

$$\int \frac{1}{1-u^2}\,du = \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| + C$$

$$\int \frac{1}{u\sqrt{1-u^2}}\,du = -\operatorname{sech}^{-1}|u| + C$$

$$\int \frac{1}{u\sqrt{1+u^2}}\,du = -\operatorname{csch}^{-1}|u| + C$$