Math 1300: Calculus I, Fall 2006; Instructor: Dana Ernst
Section 5.3: More on Curve Sketching (Part II)

Question: If $f(x) = p(x)/q(x)$ is a rational function such that $p(x)$ and $q(x)$ have no factors in common (i.e., the "fraction" is reduced), then when will $f(x)$ have a horizontal asymptote? When will it not?

Answer:

When the degree of the numerator is _________ than the degree of the denominator, other kinds of asymptotes are possible: *oblique/slant*, *curvilinear*. To see what these new kinds of asymptotes are, we use polynomial long division.

Example 1: Sketch the graph of the following function.

$$f(x) = \frac{x^2 + 1}{x}$$

Example 2: Identify the curvilinear asymptote of the following function.

$$g(x) = \frac{x^3 - x^2 - 8}{x - 1}$$

Theorem: A rational function cannot have both a horizontal asymptote and a curvilinear (including slant) asymptote. Why?

OK, we've dealt with polynomials and rational functions, how about some others? To sketch graphs of functions that are more complicated than rational functions, we follow the same set of guidelines that we wrote down in the first half of Section 5.3.

Example 2: Sketch the graphs of the following functions. (If there is not enough room on this page, use another sheet of paper.)

(a) $f(x) = 6x^{1/3} + 3x^{4/3}$

(b) $g(x) = e^{-x^2/2}$ (What is this graph?)

(c) $h(x) = x + \sin(x)$