Section 6.2 (a) *Answer:* $\dfrac{x^2}{2} + 2x - 2\ln|x+1| + C$.

(b) *Solution:* To prove the formulas, just use the definition of antiderivative. For Part i, show that

$$\frac{d}{dx}\left[\frac{\sqrt{x^4-1}}{x} + C\right] = \frac{x^4+1}{x^2\sqrt{x^4-1}}.$$

For Part ii, show that

$$\frac{d}{dx}\left[\frac{\sin 3x}{3\cos 3x} + C\right] = \frac{1}{3\cos^2 3x}.$$

Section 6.3 (a) *Answer:* $\dfrac{x^2}{2} + 5\ln|x| + C$.

(b) *Answer:* $\dfrac{1}{2}\ln(x^2+5) + C$.

(c) *Solution:* For $u = \sec(x)$ we get

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2} + C.$$

For $u = \tan(x)$ we get

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan^2(x)}{2} + C = \frac{\sec^2(x)-1}{2} + C = \frac{\sec^2(x)}{2} - \frac{1}{2} + C = \frac{\sec^2(x)}{2} + C.$$

(d) *Answer:* $\dfrac{1}{11}\cos^{11}(x) - \dfrac{1}{9}\cos^9(x) + C$.

Section 6.6 (a) *Answer:* $f(t)$ needs to be continuous on the interval $[x, x+h]$.

The MVT for integrals gives us that $\dfrac{1}{h}\displaystyle\int_x^{x+h} f(t)\,dt = f(x^*)$ for some number x^* in the interval $[x, x+h]$.

(b) *Answer:* If f is odd, the integral is 0. If f is even, the integral equals $2\displaystyle\int_0^a f(t)\,dt$.

(c) *Answer:* The integral is 0 since $\dfrac{x}{\sqrt{1+x^4}}$ is an odd function that is being integrated over a symmetric interval.

(d) *Answer:* $F'(x) = 2x\ln(x^6)$.

(e) *Answer:* -4.

Section 6.8 (a) *Solution:* We let $u = 2x + e$ so that $du = 2\,dx$. Then

$$\int_0^e \frac{dx}{2x+e} = \frac{1}{2}\int_e^{3e} \frac{du}{u} = \frac{\ln(3)}{2}.$$

(b) *Solution:* We let $u = x + 1$ so that $du = dx$. Then we have that $x = u - 1$ and hence $4x - 1 = 4(u-1) - 1$. Thus

$$\int_0^2 (4x-1)(x+1)^3\,dx = \int_1^3 (4u-5)u^3\,du = \frac{468}{5}.$$

(c) *Answer:* $\frac{1}{2}$.

(d) *Solution:* If we let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}\,dx$. Hence

$$\begin{aligned} \int_1^9 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx &= 2\int_1^9 (\cos\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)\,dx \\ &= 2\int_1^3 \cos u\,du \\ &= 2\,[\sin u]_1^3 \\ &= 2(\sin 3 - \sin 1). \end{aligned}$$

Section 7.1 (a) *Answer:* $\int_0^1 (x^2 - x^3)\,dx = \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big]_0^1 = \frac{1}{12}$.

(b) *Answer:* 4.

(c) *Answer:* The area is $A = \frac{\sqrt{8}}{3}$, and it represents the difference between the change in the velocities of the two vehicles over the time interval $[0, 2]$.

Section 7.2 (a) *Answer:* The volume is $V = \pi(1 - \frac{1}{n})$, and $V \to \pi$ as $n \to \infty$.

(b) *Answer:* For each x in $[-1, 1]$ the length of one side of a square cross-section is $2\sqrt{1-x^2}$. The volume is $\frac{16}{3}$ cubic units.

(c) *Answer:* $\frac{9}{70}$.

Section 7.3 (a) *Solution:*

i. $V(R_1) = \int_a^b \pi(f(x)^2 - g(x)^2)\,dx$;

$V(R_2) = \int_c^d 2\pi y(G(y) - F(y))\,dy$;

$V(R_3) = \int_0^s 2\pi y(k^{-1}(y) - h^{-1}(y))\,dy = \int_0^t \pi(h(x)^2 - k(x)^2)\,dx$.

ii. $V(R_1) = \int_a^b 2\pi x(f(x) - g(x))\,dx;$

$$V(R_2) = \int_c^d \pi(G(y)^2 - F(y)^2)\,dy;$$

$$V(R_3) = \int_0^s \pi(k^{-1}(y)^2 - h^{-1}(y)^2)\,dy = \int_0^t 2\pi x(h(x) - k(x))\,dx.$$

(b) *Answer:* 229.02 .

(c) *Answer:* $6\pi^2$.

True or False?

Answers:

(a) True.
$|f(x)| \geq f(x)$ for all x in $[a, b]$, therefore the inequality for their definite integrals from a to b holds by Theorem 6.5.6(b).

(b) True.
The definite integral does not depend on the variable of integration.

(c) False.
$\int f(x)\,dx$ is a class of functions of x, while $\int f(t)\,dt$ is a class of functions of t.

(d) True.
Using the substitution $u = 2x$ (hence $du = 2\,dx$) and then replacing u by x we get that

$$\int_{a/2}^{b/2} f(2x)\,dx = \int_a^b f(u)\frac{1}{2}\,du = \frac{1}{2}\int_a^b f(x)\,dx.$$

(e) False.
The Fundamental Theorem of Calculus cannot be applied to the function $h(x) = \frac{1}{x^2}$, since h is not continuous on $[-1, 1]$.
(In fact, since $h(x) = \frac{1}{x^2}$ is not bounded on $[-1, 1]$, h is not integrable on $[-1, 1]$ by Theorem 6.5.8(b).)

(f) True.
By the Fundamental Theorem of Calculus $F(x)$ is differentiable on I, and $F'(x) = f(x)$ on I. Therefore, if $F(x) = \int_a^x f(t)\,dt$ has a relative extremum at $x = b$, then $F'(b) = 0$, and hence $f(b) = F'(b) = 0$.

(g) False.
For example, let $I = [0, 2]$, $a = 0$, let $f(x) = 0$ for all $x \in I$, and let
$g(x) = \begin{cases} 0 & \text{if } x \neq 1, \\ 1 & \text{if } x = 1, \end{cases}$ where $0 \leq x \leq 2$.
Then $\displaystyle\int_0^x g(x)\,dx = 0 = \int_0^x f(x)\,dx$ for all x in $[0, 2]$, but $g(x) = f(x)$ fails for $x = 1$ in $[0, 2]$.

(h) False.
For example, $\displaystyle\int \frac{1}{x} = \ln(|x|) + C$, and $\ln(|x|)$ is not a rational function.

(i) False. $f(x)$ could be ce^x for any constant c. For example, $f(x) = 100e^x$ works.

(j) False. Since $\sin x \leq \cos x$ on $[0, \pi/4]$ and $\sin x \geq \cos x$ on $[\pi/4, \pi]$ the area enclosed by the curves is $\displaystyle\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx = 2\sqrt{2}$.