REVIEW FOR EXAM 1
Be ready to state theorems and definitions.
The exam covers material from sections: 1.1, 1.3, 1.5, 1.6, 2.1, 2.2, and 2.3.

CONCEPTS

1. True or False? Justify your answer.

   (a) The functions $u(x) = \dfrac{\sqrt{x}(x+1)}{x^2+x}$ and $v(x) = \dfrac{1}{\sqrt{x}}$ are equal.

   (b) $f \circ g = g \circ f$ holds for arbitrary functions f and g.

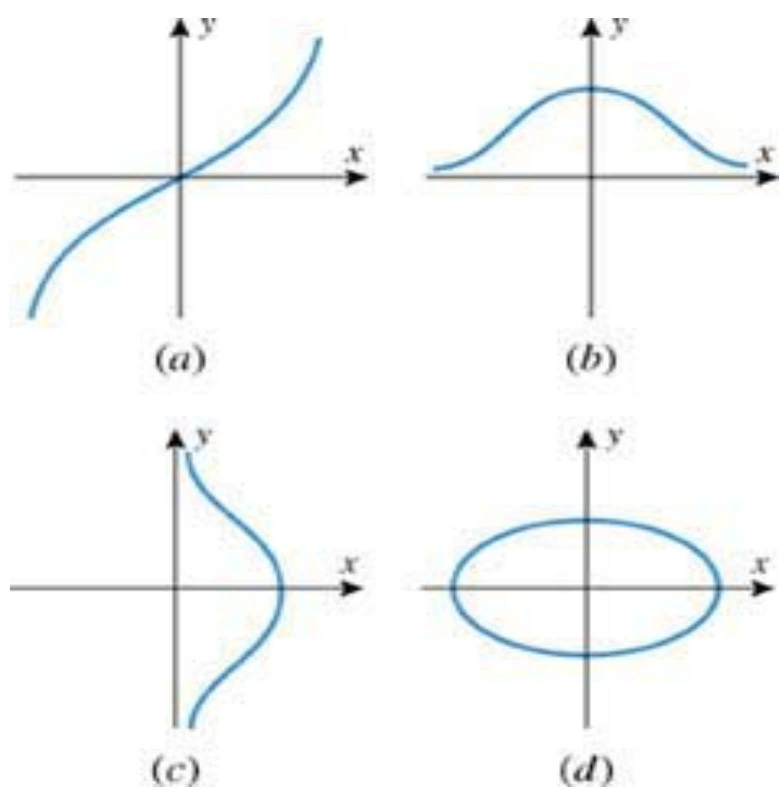
   (c) If a horizontal line does not intersect the graph of a function f, then f does not have an inverse function.

   (d) If $1 < a < b$, then $\log_a 2 > \log_b 2$.

   (e) If a function $f(x)$ does not have a limit as x approaches a from the left, then $f(x)$ does not have a limit as x approaches a from the right.

SECTION 1.1

1. Determine whether each of the following graphs is a function. If not, provide a reason why.



2. Determine if each of the following rules defines y as a function of x. If not, provide a reason why.

   (a) $x^2 + 2x + y^3 = 17$

(b) $x^2 + 2x + y^2 + 4y - 1 = 0$

(c) $x = \sin(y)$

3. Consider the following scenario.

   A set of 10 missiles get launched and each missile follows a set of directions that tell the missile what target to blow up. Some of the missiles land on the same target.

   Could this scenario represent a function? If so, identify what the function would be and what the domain and range are. If not, then explain why.

4. Consider the following scenario.

   A person types a phrase into the search engine Google, hits enter, and multiple entries are returned.

   Could this scenario represent a function? If so, identify what the function would be and what the domain and range are. If not, then explain why.

5. Consider the following function.

$$f(x) = \begin{cases} -x & \text{if } -3 \leq \text{x} \leq 0 \\ x^2 & \text{if } 0 < \text{x} < 2 \\ -1 & \text{if } 2 < \text{x} \leq 5 \end{cases} .$$

   (a) Sketch the graph of f.

   (b) What is the domain of f?

   (c) What is the range of f?

6. Consider the following function.

$$g(x) = \frac{x^2 - 4}{x + 2}$$

   (a) Sketch the graph of g.

   (b) What is the domain of g?

   (c) What is the range of g?

7. Express the following function as a single piecewise defined function.

$$f(x) = |2x - 1| - |x + 3|$$

8. A camera is mounted at a point on the ground 500 meters from the base of a space shuttle launching pad. The shuttle rises vertically when launched, and the camera's angle of elevation is continually adjusted to follow the bottom of the shuttle.

   (a) Express the height x as function of the angle of elevation θ.

(b) Find the domain of the function in part (a)

9. Determine if each of the following rules defines y as a function of x and explain why or why not. If the rule does define y as a function of x, determine the domain of the function.

   (a) $x^2 + y^2 = 3$
   (b) $x^2 + y^3 = 7$
   (c) $y(x) = \begin{cases} x^2 - 7 & \text{if } x < 0 \\ \frac{1}{x+2} & \text{if } x \geq 0 \end{cases}$
   (d) $y(x) = \begin{cases} 2x + 1 & \text{if } x \geq 1 \\ 4 - x^3 & \text{if } x \leq 1 \\ 1 & \text{if } x = 0 \end{cases}$

10. Determine the domain of the following function and write it as a piecewise defined function without using the absolute value.

   (a) $f(x) = \dfrac{x^2 + 4x - 5}{3 - |x + 2|}$

SECTION 1.3

1. Sketch the graph of the equation.

   (a) $y = \dfrac{2x - 1}{x}$
   (b) $y = -1 + \sqrt{2 - x}$

2. Let $f(x) = \sqrt{x}$ and $g(x) = x$. Does the domain of the product function $(f \cdot f)(x)$ equal the domain of $g(x)$? Justify your answer.

3. Sketch the graph of the equation $y = 3 - \dfrac{\sqrt{x - 1}}{2}$ by translating, reflecting, compressing, and/or stretching the graph of $y = \sqrt{x}$

4. Let $f(x)$ and $g(x)$ be functions such that:

   Domain of $f = (-\infty, +\infty)$
   Range of $f = (-1, 1)$
   Domain of $g = (-1, +\infty)$
   Range of $g = (-\infty, 0)$

   What is the domain of $g \circ f$?

5. If $f(x) = 8 - x$ and $g(x) = \dfrac{1}{\sqrt{x} - 1}$, find formulas for $(g \circ f)(x)$ and $(f \circ g)(x)$. What is the domain of $g \circ f$?

6. The graph of $y = a\dfrac{1}{\cos(x+b)+2} + c$ results when the graph of $y = \dfrac{1}{\cos(x)+2}$ is reflected over the x-axis, shifted 3 units to the right, and then shifted 4 units down. Find a, b, and c.

7. Find the domains of the compositions $f \circ g$ and $g \circ f$ if $f(x) = \sin^{-1} x$ and $g(x) = \ln x$.

8. Sketch the graph of the function $f(x) = \frac{1}{2}\cos^{-1}(x/3) - 1$ by translating, reflecting, stretching, compressing the graph of the inverse cosine function.

9. Sketch the graph of the function $1 - e^{2x-1}$ by translating, reflecting, stretching, compressing the graph of the natural exponential function.

SECTION 1.5

1. For each of the following functions, find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$.

   (a) $f(x) = \dfrac{x+1}{x-1}$
   (b) $f(x) = 3x^3 - 15$

2. Complete the following identities.

   (a) $\tan(\sin^{-1} x) =$
   (b) $\cos(\tan^{-1} x) =$

3. For each of the following functions, find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$.

   (a) $f(x) = \sqrt{-x}$
   (b) $f(x) = 12x^3 - 1$

SECTION 1.6

1. Simplify $\ln(x^2)$.

2. Solve for x in the following equations.

   (a) $\ln\left(x^2\right) = \ln x$
   (b) $\ln\left(x^2\right) = (\ln x)^2$
   (c) $\ln\sqrt{x} = \sqrt{\ln x}$

3. Solve for x in the following equations.

   (a) $5^x = 4$
   (b) $2^{4x+1} = 3$

4. Given the equation $2^{t/\tau} = e^{\alpha t}$, solve for the *growth constant* α in terms of τ using natural logarithms. (The parameter τ is called the *doubling time*.)

5. Solve the following equations for t (in terms of x) using natural logarithms

   (a) $\dfrac{e^t + e^{-t}}{2} = x$

   (b) $\dfrac{e^t - e^{-t}}{e^t + e^{-t}} = x$

6. Simplify the following.

   (a) $(-27)^{2/3}$

   (b) $e^{3\ln \pi}$

   (c) $\log_4 \dfrac{1}{2} + \log_4 8 + \log_4 16$

7. Solve for x.

   (a) $\ln(x^2+1) - \ln x = \ln 2$

   (b) $e^{-2x} - 4e^{-x} = 5$

8. Explain how the graph of the logarithmic function with base $b > 0$ can be obtained from the graph of the natural logarithmic function by using one or more translations, reflections, stretches or compressions.

SECTION 2.1

1. Sketch the graph of a possible function f that has all properties (a)-(g) listed below.

   (a) The domain of f is $[-1, 2]$

   (b) $f(0) = f(2) = 0$

   (c) $f(-1) = 1$

   (d) $\lim_{x\to 0^-} f(x) = 0$

   (e) $\lim_{x\to 0^+} f(x) = 2$

   (f) $\lim_{x\to 2^-} f(x) = 1$

   (g) $\lim_{x\to -1^+} f(x) = -1$

2. Sketch the graphs of possible functions f, g, and h such that: f satisfies property (a) below, g satisfies property (b) below, and h satisfies property (c) below. There should be three separate graphs.

   (a) $\lim_{x\to 0} f(x) = 1$

   (b) $\lim_{x\to 0^-} g(x) = -1$ and $\lim_{x\to 0^+} g(x) = +1$

   (c) $\lim_{x\to 0} h(x) \neq h(0)$

3. Find an equation for the tangent line to the curve $y = x^3$ at the point $(1, 1)$.

SECTION 2.2

1. Find the following limits.

   (a) $\lim_{x\to 2} x^2+4x-12$

   (b) $\lim_{x\to 2} \dfrac{x^2+4x-12}{x^2+4x+3}$

   (c) $\lim_{x\to 2} \dfrac{x^2+4x-12}{x^2-x-2}$

   (d) $\lim_{x\to 2} \dfrac{x^2+4x-12}{x^2-4x+4}$

   (e) $\lim_{x\to -3} \dfrac{x}{x+3}$

   (f) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$

2. For f defined as follows, find the given limits:

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ 2x+1 & \text{if } 0 \le x \le 4 \\ x^2 & \text{if } x > 4 \end{cases}$$

   (a) $\lim_{x\to 0^+} f(x)$

   (b) $\lim_{x\to 4} f(x)$

3. Let $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 6$, and $\lim_{x\to a} h(x) = 0$. Find the following limits, if they exist:

   (a) $\lim_{x\to a}(f(x)+2g(x))$

   (b) $\lim_{x\to a} \dfrac{(g(x))^2}{f(x)+5}$

   (c) $\lim_{x\to a} \dfrac{2f(x)}{h(x)}$

   (d) $\lim_{x\to a} \dfrac{7f(x)}{2f(x)+g(x)}$

   (e) $\lim_{x\to a} \sqrt[3]{g(x)+2}$

4. Find the following limits:

   (a) $\lim_{x\to -2} \dfrac{x^2-5x-14}{x+2}$

   (b) $\lim_{x\to 0} \dfrac{4x-3}{4x^2+3}$

(c) $\displaystyle\lim_{x\to 3^-} \frac{x^2+3x+2}{x^2-2x-3}$

SECTION 2.3

1. Find the following limits.

   (a) $\displaystyle\lim_{x\to\infty} 5x^2-2x+1$

   (b) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{3x^2-5}}{x-7}$

   (c) $\displaystyle\lim_{x\to\infty} \frac{2}{\pi}\tan^{-1}(x)$

   (d) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{5x^2-2}}{x+3}$

   (e) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+3}-x$

   (f) $\displaystyle\lim_{x\to 1^-} \ln(1-x)$