Math 1300: Calculus I, Fall 2006
Review for Midterm Exam 2

Here is a review for your upcoming exam. The exam covers sections 2.5 through 4.3 of our textbook. You should be prepared to state definitions and theorems.

I. True or False? Justify your answers.

(a) The function $f(x) = x\cos\left(\frac{1}{x}\right)$ is continuous everywhere.

(b) If a function is not continuous at $x = c$, then either it is not defined at $x = c$ or it does not have a limit as x approaches c.

(c) If a function is not continuous at $x = c$, then it is not differentiable at $x = c$.

(d) If $f(x)$ is a function such that $f(0) < 0$ and $f(2) > 0$, then there is a number c in the interval $(0, 2)$ such that $f(c) = 0$.

(e) If $h(x) \leq f(x) \leq g(x)$ for all real numbers x and $\lim_{x\to c} h(x)$ and $\lim_{x\to c} g(x)$ exist, then $\lim_{x\to c} f(x)$ also exists.

(f) If $f(x)$ is differentiable at $x = 0$, then $\lim_{h\to 0} \frac{f(h)}{h}$ exists.

(g) The derivative with respect to x of a rational function $f(x)$ is a rational function.

(h) $\frac{d}{dx}[f(cx)] = c\frac{d}{dx}[f(x)]$.

(i) $\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{1}{f'(x)}$.

II. Recommended Problems from the Textbook

Section 2.5: Quick Check 2, 23(a)(b), 28, 29(a)(b)(c), 38

Section 2.6: Quick Check 4, 7, 14, 21, 25, 27, 33, 37, 46, 47

Section 3.1: 9, 21

Section 3.2: 23, 25, 41

Section 3.3: 22, 23, 43, 47, 51, 53, 55, 63

Section 3.4: 13, 19, 21, 25

Section 3.5: 11, 13, 16, 22, 23, 25, 29, 31, 35, 36, 39, 44

Section 3.6: 14, 21, 32, 43, 54, 57

Section 3.7: 23, 37, 41

Section 4.1: 19, 24, 29, 33, 49

Section 4.2: 38(b), 43, 44

Section 4.3: 29, 41, 43, 47, 61, 67, 71

III. Additional Problems

1. Find the values of x, if any, for which the following function is not continuous.

$$f(x) = \begin{cases} \frac{x}{x^2-1}, & x \geq 0 \\ \frac{x+3}{|x-3|}, & x < 0. \end{cases}$$

2. Find the values for k such that the following function $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} \sqrt{x^2-16}, & x \geq 5 \\ \frac{3k}{x-1}, & x < 5. \end{cases}$$

3. Is the following function continuous on the interval $[0,1]$? Justify your answer.

$$f(x) = \frac{1}{x^5 + \pi x - e}$$

4. If $3x \leq f(x) \leq x^3 + 2$ for $0 \leq x \leq 2$, evaluate $\lim_{x\to 1} f(x)$.

5. Prove that $\lim_{x\to 0} \left(x^4 \cos(2/x)\right) = 0$.

6. A representative from a certain publishing company is dropped from the top of a 220 foot tall building. The height $h(t)$ in feet of the representative at t seconds is given by the position function $h(t) = -16t^2 - 26t + 220$.

   (a) Find the average velocity of the representative between 2 seconds and 4 seconds. Label your answer with the appropriate units.

   (b) Find the instantaneous velocity of the representative at 2 seconds. Label your answer with the appropriate units.

   (c) What does the sign of your answer in part (b) mean?

7. Let $f(x) = x^2 - x$.

   (a) Find $f'(x)$ using the limit definition of the derivative.

   (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

8. Let $y = x^2 \sin\left(\frac{\pi}{4}x\right)$. Find $\frac{dy}{dx}$.

9. True or False?

$$\frac{d}{dx}[(2x^3 + 5x^2 - 7)(3\cos(x) + 13x)] = (6x^2 + 10x)(-3\sin(x) + 13)$$

Justify your answer.

10. Find the derivative of $\csc(\csc(\sin(x)))$

11. Find $\frac{d}{dx}[\sec(x)\tan(x) + (\cos(x))^2]$

12. Find the equation for the line tangent to the graph of $f(x) = \csc(2x) + \tan(x)$ at the point $(\frac{\pi}{4}, 2)$

13. Prove that $\frac{d}{dx}[\cos(x)] = -\sin(x)$ using the definition of a derivative as a limit. The following identities might be helpful.

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\lim_{a\to 0} \frac{\sin(a)}{a} = 1$$

$$\lim_{a\to 0} \frac{1-\cos(a)}{a} = 0$$

14. If $f(1) = 3$, $g(5) = 2$, $h(1) = 5$, $f'(2) = 3$, $g'(5) = -2$, and $h'(1) = -8$, find $(f \circ g \circ h)'(1)$

15. A spherical balloon is deflated so that its volume decreases at a constant rate of 3 in^3/sec. How fast is the balloon's diameter decreasing when the radius is 2 inches?

16. A 10-foot ladder is leaning against a building. If the top of the ladder slides down the wall at a constant rate of 2 feet per second, how fast is the acute angle the ladder makes with the ground decreasing when the top of the ladder is 5 feet from the ground? (Give the answer in radians per second.)

17. A person 6 ft tall walks at 5 ft/s along the edge of a road 30 ft wide. On the other edge of the road is a light atop a pole 18 ft high. How fast is the length of the person's shadow (on the horizontal ground) increasing when the person is 40 ft from the point directly across the road from the pole?

18. (a) Compute $\frac{dy}{dx}[\sin^{-1}(x)]$ using implicit differentiation.
    (b) Find the equation of the tangent line at $c = \frac{1}{2}$.

19. Differentiate each of the following functions.
    (a) $f(x) = e^{e^{e^{e^x}}}$.
    (b) $f(x) = (\log x)^{\log x}$.

20. Let $f(x) = \frac{x}{x^2+3}$.
    (a) Use the quotient rule to calculate $f'(x)$
    (b) Calculate $f'(x)$ using the product and the chain rule (Hint: Rewrite $f(x)$ as $f(x) = x(x^2+3)^{-1}$.)
    (c) Find $f'(x)$ using logarithmic differentiation.

21. Find $\frac{dy}{dx}$ for each of the following:

    (a) $y = \ln x^2$

    (b) $y = x \ln 2$

    (c) $y = x^e$

    (d) $y = x^x$

    (e) $y = 2^x$

    (f) $y = \ln(\cos x)$

    (g) $y = e^{3x+5}$

    (h) $y = (11x^2 + 9x - 7)^{(\ln x)}$ (Hint: Use logarithmic differentiation.)

    (i) $y = \left(x^{\ln x}\right)^{\ln x}$

22. Use logarithmic differentiation to find $f'(x)$ for each of the following.

    (a) $f(x) = \frac{(3-x)^{1/3}x^2}{(1-x)(3+x)^{2/3}}$.

    (b) $f(x) = (x+1)(e^{x^2} + 1)$