Math 1300: Calculus I, Fall 2006
Review for the Final Exam

The Final exam will cover sections 1.1, 1.3–1.6, 2.1–2.3, 2.5–2.6 3.1–3.7, 4.1–4.4, 5.1–5.5, 5.7, 6.1–6.6, 6.8, and 7.1–7.3. You should be prepared to state definitions and theorems on the exam.

Do the following problems as part of your review of the material covered after the last midterm exam. To review material covered earlier, use the problems on the review sheets for the midterm exams.

Section 6.2 (a) Find $\displaystyle\int \frac{x^2+3x}{x+1}\,dx$.

(b) Prove the following formulas:

i. $\displaystyle\int \frac{x^4+1}{x^2\sqrt{x^4-1}}\,dx = \frac{\sqrt{x^4-1}}{x} + C,$

ii. $\displaystyle\int \frac{1}{\cos^2 3x}\,dx = \frac{\sin 3x}{3\cos 3x} + C.$

(c) Book Problems: 9, 13, 15, 23, 25, 27, 33, 39, 43, 45.

Section 6.3 (a) Find $\displaystyle\int \frac{x^2+5}{x}\,dx$.

(b) Find $\displaystyle\int \frac{x}{x^2+5}\,dx$.

(c) Find $\displaystyle\int \sec^2(x)\tan(x)\,dx$ using u-substitution in two different ways; first use $u = \sec(x)$ and then $u = \tan(x)$.

(d) Evaluate $\displaystyle\int \sin^3(x)\cos^8(x)\,dx$.

(e) Book Problems: 9, 25, 31, 35, 43, 51, 59.

Section 6.6 (a) In order to use the Mean-Value Theorem for integrals of the form $\displaystyle\frac{1}{h}\int_x^{x+h} f(t)\,dt$ some assumptions need to be made about $f(t)$. What are these assumptions? Write down what the MVT for integrals gives us for an integral of this form.

(b) If $f(x)$ is and odd function then what can be said about an integral of the form $\displaystyle\int_{-a}^{a} f(x)\,dx$? What if $f(x)$ is even? Explain.

(c) Evaluate: $\int_{-1}^{1} \frac{x}{\sqrt{1+x^4}}\, dx$.

(d) Compute $F'(x)$ for $F(x) = \int_1^{x^2} \ln(w^3)\, dw$.

(e) Evaluate: $\int_0^2 (-3x^3 + 3x^2 - x + 1)\, dx$.

(f) Book Problems: 23, 29, 39, 41, 57, 61.

Section 6.8 (a) Evaluate the following definite integral: $\int_0^e \frac{dx}{2x+e}$.

(b) Evaluate the following definite integral: $\int_0^2 (4x-1)(x+1)^3\, dx$.

(c) Evaluate the following: $\int_{\frac{\pi}{2}}^{\pi} 6\sin(x)[\cos(x)+1]^{11}\, dx$.

(d) Evaluate $\int_1^9 \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$.

(e) Book Problems: 15, 23, 37, 41.

Section 7.1 (a) Compute the area of the region enclosed by the graphs of the following two functions: $f(x) = x^2$ and $g(x) = x^3$.

(b) Graph and find the area between $y = \sin(x)$ and $y = \cos(x + \frac{\pi}{2})$ between 0 and π.

(c) The following are two acceleration functions, with respect to time, for two different vehicles. $A_1(t) = \sqrt{t}, A_2(t) = \frac{1}{\sqrt{8}}t^2$. Find the area enclosed by the two acceleration curves and explain what it represents.

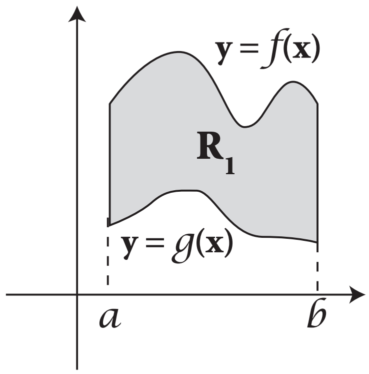
(d) Book Problems: 5, 13, 17, 21, 47.

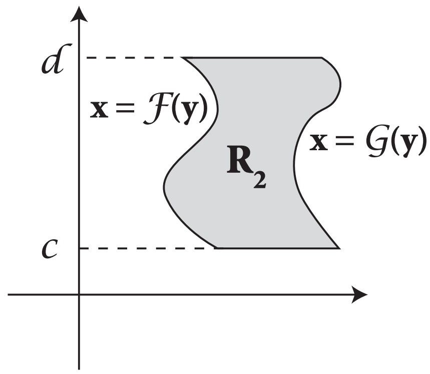
Section 7.2 (a) Let n be a real number greater than 1. Find the volume of the solid formed by revolving the plane region bounded by $x = 1$, $y = \frac{1}{x}$, $x = n$, and $y = 0$ about the x-axis. (Your answer should be in terms of n.) What is the limit of the volume as $n \to \infty$?

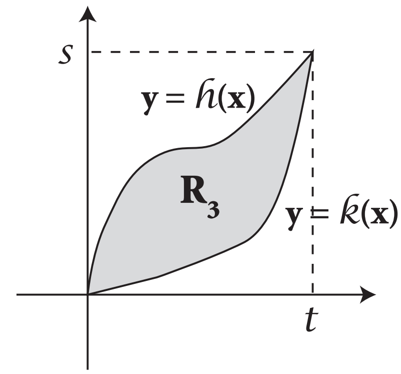
(b) Find the volume of the solid formed whose base is the disk $x^2 + y^2 \leq 1$ and whose cross sections perpendicular to the x-axis are squares.
Hint: Find the volume by slicing with the squares as your slices.

(c) Find the volume of the solid whose base is the region bounded between the curves $y = \sqrt{x}$ and $y = x^2$, and whose cross sections perpendicular to the x-axis are squares.

(d) Book Problems: 13, 25, 27, 29, 55.







Section 7.3 (a) For each region above, write an integral expression for the volume of the solid generated by

i. rotating the region around the x-axis,

ii. rotating the region around the y-axis.

(b) Use cylindrical shells to find the volume of solid generated when the region in the first quadrant enclosed between $\sqrt{x}$ and $\frac{1}{27}x^2$ is revolved about the y-axis. Round your answer to the nearest hundredth.

(c) Find the volume of the inner tube that is 8 feet wide and 2 feet tall viewed from the side and has a hole of 4 foot diameter in the middle when viewed from the top.
Hint: Let the y-axis run through the middle of the hole and let the x-axis cut the inner tube in half, viewed from the side. Think of the tube as a region rotated around the y-axis. You may need a formula from geometry when integrating.

(d) Book Problems: 3, 5, 11, 23, 27.

True or False? Justify your answer. (For questions (a)–(g) assume that f is continuous on an interval I and that a, b are in I.)

(a) $\displaystyle\int_a^b |f(x)|\,dx \geq \int_a^b f(x)\,dx.$

(b) $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(t)\,dt.$

(c) $\displaystyle\int f(x)\,dx = \int f(t)\,dt$ on I.

(d) $\displaystyle\int_{a/2}^{b/2} f(2x)\,dx = \frac{1}{2}\int_a^b f(x)\,dx.$

(e) By the Fundamental Theorem of Calculus,

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big]_{-1}^{1} = -1 - (+1) = -2.$$

(f) If the function $F(x) = \displaystyle\int_a^x f(t)\,dt$ has a relative extremum at $x = b$, then $f(b) = 0$.

(g) If g is an arbitrary integrable function such that $\displaystyle\int_a^x g(x)\,dx = \int_a^x f(x)\,dx$ for all x in I, then $g(x) = f(x)$ for all x in I.

(h) The antiderivatives of rational functions are rational functions.

(i) If f is continuous everywhere, a is a constant, $F(x) = \displaystyle\int_a^x f(t)\,dt$, and $F'(x) = f'(x)$ for all x, then $f(x) = e^x$ or $f(x) = 0$.

(j) The area enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi$ is $\displaystyle\int_0^{\pi} (\sin x - \cos x)\,dx = 2$