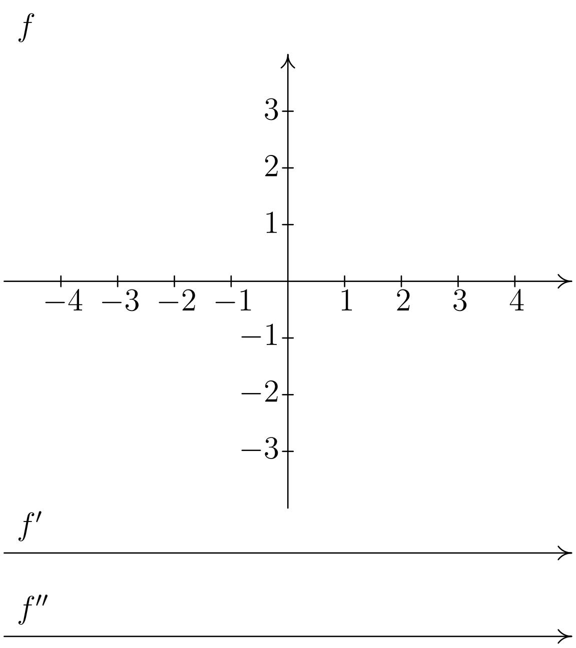
MA 2550: Calculus I

Section 4.5: Summary of Curve Sketching (Part 1)

In general, we will be given a function and asked to sketch its graph. To do this, we will have to identify some key features of the graph (see guidelines below). Let's first try to sketch the graph of a function where some useful information is given.

Example 1: Sketch the graph of the function that has the following properties.

1. $f(-5) = 0, f(-3) = -3, f(-2) = 0$
2. $f(-1.5) = .5, f(-.5) = 1, f(1.5) = 2.5$
3. $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 3} f(x) = \infty$
4. $\lim_{x\to \infty} f(x) = 1$ and $\lim_{x\to -\infty} f(x) = 1$
5. $f'(-3)$ undefined
6. $f'(1.5) = 0, f'(-1.5) = 0$
7. $f'(x) > 0$ on $(-3, -1.5)$, $(-1.5, 0)$, and $(1.5, 3)$
8. $f'(x) < 0$ on $(-\infty, -3), (0, 1.5)$, and $(3, \infty)$
9. $f''(x) > 0$ on $(-1.5, 0), (0, 3)$, and $(3, \infty)$
10. $f''(x) < 0$ on $(-\infty, -3)$ and $(-3, -1.5)$



Guidelines for Sketching Graphs of Functions The following checklist is intended as a guide to sketching a curve $y = f(x)$ by hand. Not every item is relevant to every function.

1. Consider Domain.
2. Determine whether there is symmetry about the y-axis or the origin.
3. Find x and y-intercepts. (Finding x-intercepts is not always easy and finding them should be abandoned if too difficult.)
4. Identify vertical asymptotes.
5. Determine end behavior by computing limits of $f(x)$ as $x \to \infty$ and $x \to -\infty$ (Does graph have any horizontal or slant asymptotes?).
6. Find critical points, determine intervals of increase and decrease, and identify any relative extrema.
7. Find x-values where $f''(x) = 0$ or is undefined, determine intervals of concavity, and identify any inflection points

For now, we won't encounter any slant asymptotes. More on that next time.

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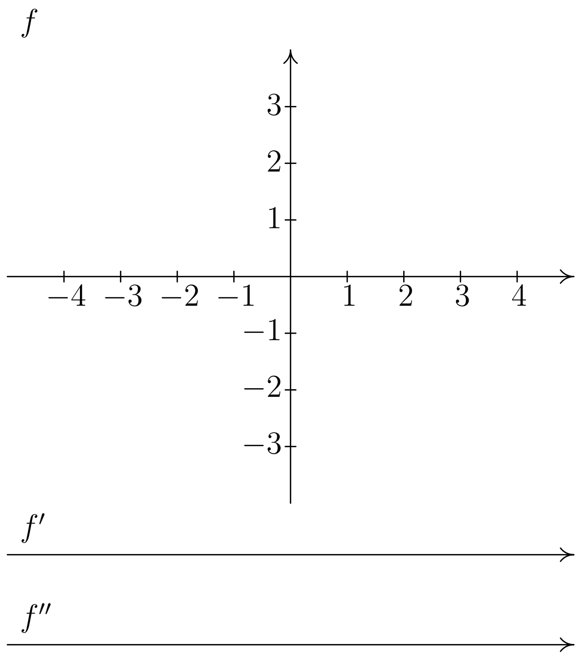
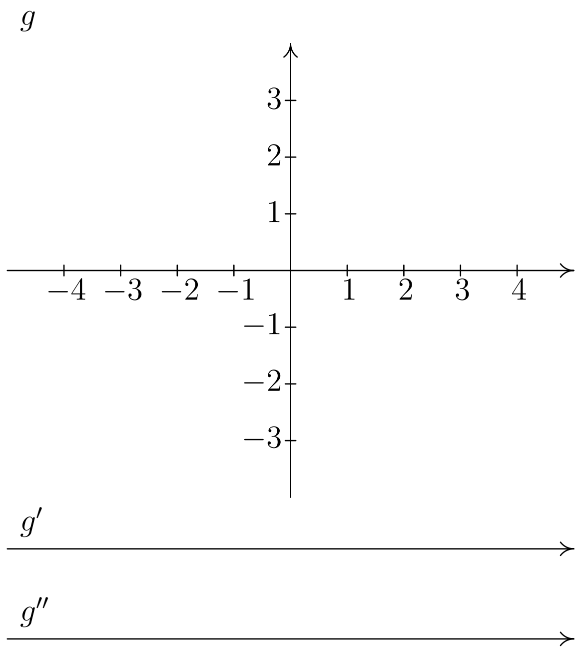
Example 2: Sketch the graph of the following functions.

(a) $f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$



(b) $g(x) = \dfrac{-x}{(x^2-1)^2}$



(c) $h(x) = x^{5/3} - 5x^{2/3}$