Plymouth State University

MA 2550: Calculus I

Review for Exam 2

Exam 2 will be on **Tuesday, October 28**. Exam 2 covers material from sections 3.3–3.6, 3.8, 3.9, 4.1, and 4.2. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions on this review. I will not collect this review and you may do what you want with it.

TOPICS

Here is a list of topics/concepts that you must have an understanding of or be able to do:

- be able to find derivatives of functions using differentiation rules
- in particular, be able to apply the Power Rule, Product Rule, Quotient Rule, and Chain Rule appropriately
- know derivatives of $\sin x$, $\cos x$, $\tan x$, and $\sec x$
- know the limits involving sine and cosine that appear in the red boxes on pages 149 and 150, respectively
- understand implicit differentiation and be able to find derivatives of implicit functions
- be able to solve related rates word problems (you should be able to label your answers with the appropriate units)
- be able to find the equation of the tangent line to the graph of a function at a particular point
- be able to use the tangent line at a particular point to approximate the value of a function
- be able to find differentials
- understand how dy, dx, Δx, and Δy are all related
- be able approximate the change in a function over a particular interval using differentials
- know definition of critical number and be able to find them
- understand the concept of local max/min
- know the relationship between critical numbers and local max/min
- understand the concept of absolute max/min
- know the statement of and understand the Extreme Value Theorem
- be able to find absolute max/min of a continuous function on a closed interval
- know the statement of and understand Rolle's Theorem
- know the statement of and understand the Mean Value Theorem

- be able to find the points guaranteed to exist according to the Mean Value Theorem
- be prepared to provide examples that illustrate various concepts (for example, you should be able to state an example of a function that has a critical number, but does not have a local max or local min)

WORDS OF ADVICE

Here are somethings to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show how and why you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

EXERCISES

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
   (a) If a function is not continuous at $x = c$, then it is not differentiable at $x = c$.
   (b) The derivative with respect to x of a rational function $f(x)$ is a rational function.

(c) $\dfrac{d}{dx}[f(cx)] = c\dfrac{d}{dx}[f(x)]$.

(d) $\dfrac{d}{dx}\left[\dfrac{1}{f(x)}\right] = \dfrac{1}{f'(x)}$.

(e) $\dfrac{d}{dx}[(2x^3 + 5x^2 - 7)(3\cos(x) + 13x)] = (6x^2 + 10x)(-3\sin(x) + 13)$

(f) If $f'(c) = 0$, then f has a local max or local min at $x = c$.

(g) If f has a local max at $x = c$, then $f'(c) = 0$.

2. Differentiate each of the following functions, but do *not* simplify.

(a) $f(x) = \pi^2$

(b) $g(x) = \dfrac{x}{3} + \sqrt{x} - \dfrac{1}{x} + \sqrt{2}$

(c) $y = x^3\sqrt{x+1}$

(d) $h(x) = \sin(\sin x)$

(e) $k(x) = \sin^2 x$

(f) $f(x) = \sin x^2$

(g) $r(x) = \dfrac{x^2 - 3x + 1}{2 - x}$

(h) $y = \dfrac{3x - 2x^2}{x^{2/3}}$

(i) $f(x) = 2\cos x + \sec 2x$

(j) $g(x) = \dfrac{1 + \cos x}{1 - \cos x}$

(k) $\tan^3(1 - 5x^2)$

(l) $y = x^2 \sin\left(\dfrac{\pi}{4}x\right)$

3. If $f(1) = 3$, $g(5) = 2$, $h(1) = 5$, $f'(2) = 3$, $g'(5) = -2$, and $h'(1) = -8$, find $(f \circ g \circ h)'(1)$

4. Exercise 63(a) on page 162

5. For each of the following, find dy/dx.

(a) $x^2y + y^2 = x$

(b) $x = \sin 2y$

6. Find the equation of the tangent line to the graph of $f(x) = 2\sin x$ when $x = \pi/3$.

7. Find the equation of the tangent line to the graph of $g(x) = \sqrt{x-3}$ when $x = 7$.

8. Find the equation of the tangent line to the graph of $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

9. A spherical balloon is deflated so that its volume decreases at a constant rate of 3 in^3/sec. How fast is the balloon's diameter decreasing when the radius is 2 inches?

10. A 10-foot ladder is leaning against a building. If the top of the ladder slides down the wall at a constant rate of 2 feet per second, how fast is the acute angle the ladder makes with the ground decreasing when the top of the ladder is 5 feet from the ground? (Give the answer in radians per second.)

11. An interstellar nugget is circling the planet Earth a mile from its surface in a clockwise direction. An observer is standing at a fixed location watching the nugget with his nuggetscope. If the nugget is traveling at a rate of 200 mph, then what is the rate of change in the angle of elevation of the nuggetscope when the angle is $\pi/4$? For this problem, assume the surface of the Earth is locally flat (not round) and assume that the observer is standing facing the nugget with the nugget approaching.
12. Approximate the value of $\sqrt[3]{26}$ using linear approximation.
13. Exercise 33(a) on page 194
14. Find all critical numbers of $f(x) = x\sqrt{4 - x^2}$.
15. Exercise 11 on page 211
16. Find the absolute max and min for $f(x) = \cos x - x$ on the interval $[0, 2\pi]$.
17. Find the absolute max and min for $g(x) = 2x + \dfrac{1}{2x}$ on the interval $[1, 4]$.
18. Determine whether Rolle's Theorem applies to each of the following functions on the indicated interval. Explain your answer.
    (a) $f(x) = \dfrac{x^3 + x}{x}$, $[-1, 1]$
    (b) $g(x) = \dfrac{x^2}{x^2 - 3}$, $[-3/2, 3/2]$
19. Given $f(x) = 10 - \dfrac{16}{x}$, show that f satisfies the hypotheses of the Mean Value Theorem on the interval $[2, 8]$, and then find all numbers c that the Mean Value Theorem guarantee exist.
20. Exercise 32 on page 220
21. Exercise 33 on page 220