MA 3110: Logic, Proof, and Axiomatic Systems

Guidelines for Exam 2

Exam 2 covers material in sections 1.6 and 2.1–2.4 of our textbook, as well as any material discussed in class and any material from Exam 1. In fact, one question on the in-class portion of Exam 2 will be identical or nearly identical to a question from Exam 1. As with Exam 1, Exam 2 will consist of two parts: an in-class part and a take-home part.

Part I: In-class exam

The in-class part of Exam 2 will take place on **Wednesday, November 5**. This portion of the exam will test your knowledge of definitions and basic concepts. You should be prepared to generate examples (for example, "provide an example of two sets whose intersection is nonempty" or "provide an example of a nonempty family of pairwise disjoint sets"). To be successful on the in-class portion of Exam 2 you should

- know how to write a direct proof of statements of the form $(\forall x)P(x)$
- know how to write a proof by contradiction of statements of the form $(\forall x)P(x)$
- know how to write a proof of statements of the form $(\exists! x)P(x)$
- for statements involving multiple quantifiers, know what manipulations of these quantifiers permit valid deductions (see pages 50–51)
- know definitions of *set*, *element*, *emptyset*, and *power set*
- know what $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are
- understand set notation of the form $\{x : P(x)\}$
- know definition of *subset* and be able to write a direct proof that $A \subseteq B$
- know definition of *equality of two sets* and be able to prove that two sets are equal (we had two methods for doing this and you should know both)
- know the statements of and be able to apply Theorems 2.1, 2.2, 2.4, and 2.5
- know and understand the definitions of *union*, *intersection*, and *set difference*
- know definition of *disjoint*
- develop intuition about the statements in Theorem 2.6 (you do not need to memorize this theorem)
- know and understand the definition of *complement* (why do we need to make reference to a universe?)
- know the statements of and be able to apply parts (e) and (f) of Theorem 2.7; develop intuition about the remaining parts of this theorem

- know definition of a *family of sets*
- understand what an *indexed family of sets* is
- know and understand definitions of *union* and *intersection* over a family of sets (indexed or not)
- develop intuition about the statements in Theorem 2.8
- know the statements of and be able to apply parts (c) and (d) of Theorem 2.9; develop intuition about the remaining parts of this theorem
- know definition of *pairwise disjoint*
- know statement of the PMI
- know definition of *inductive set*
- be able to write a proof by induction of statements of the form $(\forall n \in \mathbb{N})P(n)$
- be able to evaluate the validity of a proposed "proof" of a statement involving relevant definitions
- as well as being able to generate example, you should be able to construct counterexamples to show that a given statement is false

Additionally, you should be able to call upon your own prodigious mental faculties to respond in flexible, thoughtful, and creative ways to problems that may seem unfamiliar on first glance. (Humans are awesome - I don't care what Doron Zeilberger says.) Finally, you should prepare yourself sufficiently that you can read and understand without undue anxiety.

Part 2: Take-home exam

As with the previous exam, the take-home portion of Exam 2 will consist of 5 theorems and you will be required to prove any 3 of them. This half of Exam 2 is due to my office (Hyde 312) by 5 pm on Friday, November 7th (no exceptions). These are the simple rules for the take-home portion of the exam:

1. You are NOT allowed to copy someone else's work.
2. You are NOT allowed to let someone else copy your work.
3. You are allowed to discuss the problems with each other and critique each other's work.

I was slightly disappointed in the blatant copying that occurred on the previous exam. Please, just follow these extremely generous rules and don't screw it up for everyone else.