MA 3110: Logic, Proof, and Axiomatic Systems (Fall 2008)

EXAM 2 (Take-home portion)

NAME:

Instructions: These directions are slightly different than for Exam 1; so, **read this entire page**. Prove THREE of the following theorems, where *at least one* of theorems proven is Theorem 4 or Theorem 5. You may choose to prove both Theorem 4 and Theorem 5, but you don't have to. I expect your proofs to be well-written, neat, and organized. You should write in complete sentences. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

This portion of Exam 2 is worth 30 points, where each proof is worth 10 points.

These are the simple rules for this portion of the exam:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using.
2. You are NOT allowed to copy someone else's work.
3. You are NOT allowed to let someone else copy your work.
4. You are allowed to discuss the problems with each other and critique each other's work.

If these simple rules are broken, then the remaining exams will be all in-class with no reduction in their difficulty.

This half of Exam 2 is due to my office (Hyde 312) by **5 pm on Friday, November 7th** (no exceptions). You should turn in this cover page and the three proofs that you have decided to submit.

Good luck and have fun!

Theorem 1: Let A, B, C be sets. If $A \subseteq B \cup C$ and $A \nsubseteq B$, then $A \cap B \neq \emptyset$.

Important: In its current form, this "theorem" is false. The conclusion was supposed to be $A \cap C \neq \emptyset$. You may either provide a counterexample to show that the original statement is false or you may prove the corrected statement.

Theorem 2: Let A and B be sets in a universe U. Then $A \cup B^c = U$ iff $B \subseteq A$.

Theorem 3: Let A, B, C be sets. If $C \subseteq A \cap B$, then $A \subseteq (A - C) \cup B$.

Theorem 4: For every $n \in \mathbb{N}$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$.

For the next theorem, you must prove both parts. Prove part (a) by induction.

Theorem 5: Let $\{A_n : n \in \mathbb{N}\}$ be a family of sets satisfying $A_n \subseteq A_{n+1}$ for all $n \geq 1$.

(a) For all $n \in \mathbb{N}$, $A_1 \subseteq A_n$.

(b) $\displaystyle\bigcap_{n=1}^{\infty} A_n = A_1$.