Plymouth State University

MA 3110: Logic, Proof, and Axiomatic Systems (Fall 2008)

EXAM 3 (Take-home portion)

NAME:

Instructions: Prove any THREE of the following theorems. If you turn in more than three proofs, I will only grade the first three that I see. I expect your proofs to be well-written, neat, and organized. You should write in complete sentences. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

This portion of Exam 3 is worth 30 points, where each proof is worth 10 points.

The usual rules apply. The simple rules for this portion of the exam are:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using.
2. You are NOT allowed to copy someone else's work.
3. You are NOT allowed to let someone else copy your work.
4. You are allowed to discuss the problems with each other and critique each other's work.

This half of Exam 3 is due to my office (Hyde 312) by **5 pm on Friday, December 5th** (no exceptions). You should turn in this cover page and the three proofs that you have decided to submit.

Good luck and have fun!

Theorem 1: Let f_n denote the n^{th} Fibonacci number.[1] Then $f_n \leq 2^n$ for all $n \geq 1$.

[1] Recall that the Fibonacci numbers are defined by $f_1 = f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

Theorem 2: For any sets A, B, C, and D, if $(A \times B) \cap (C \times D) = \emptyset$, then $A \cap C = \emptyset$ or $B \cap D = \emptyset$.

Theorem 3: Let $X = \mathbb{N} \times \mathbb{N}$ and define the relation $\sim$ on X via

$$(x, y) \sim (z, w) \text{ iff } x + w = y + z.$$

Then $\sim$ is an equivalence relation on X.

Theorem 4: Let m be a nonzero integer. Consider the equivalence relation $\equiv_m$ on $\mathbb{Z}$ as defined in the book and in class.[2] If $x \equiv_m y$ and $z \equiv_m w$, then $x + z \equiv_m y + w$.[3]

[2] You do *not* need to prove that $\equiv_m$ is an equivalence relation.

[3] This theorem allows us to define addition of equivalence classes mod m, otherwise known as modular arithmetic. You do not need to prove this, but it is worth pondering.

Theorem 5: Let X be a nonempty set and let R be an equivalence relation on X. Define $\phi : X \to X/R$ via

$$\phi(x) = x/R$$

for all $x \in X$.[4] Then xRy iff $\phi(x) = \phi(y)$.

[4]Note that ϕ is the canonical map from X to X/R.