# Lab 1: Exploring limits

**Names:**

## Goal

Develop more intuition about limits by exploring a couple of interesting limits.

## Directions

In groups of 2–4, answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). The lab is due on **Monday, September 21** and is worth 10 points. If you finish the lab in class today, you can turn it in before you leave.

## Exercises

Recall that we say "the limit of $f(x)$ as $x$ approaches $a$ is $L$" provided that we can make the values of $f(x)$ arbitrarily close to $L$ by taking $x$ to be sufficiently close to $a$ (on either side) but not equal to $a$. We write

$$\lim_{x \to a} f(x) = L.$$

Often, we can gain intuition about a limit by exploring what happens with values "near" $x = a$. As the next two examples will illustrate, sometimes this can be misleading.

1. Let $f(x) = \dfrac{\sqrt{x^2 + 9} - 3}{x^2}$. We would like to find $\lim_{x \to 0} f(x)$. We picked 0 because if you look at the formula for $f$, something interesting happens at 0 (since $f(0)$ is undefined).

    (a) Let's take a look at what happens at some $x$-values that are pretty close to 0. Using your calculator, complete the following table. (Round your answers to 5 decimal places.)

    | $x$ | $f(x)$ |
    |---|---|
    | $\pm 1$ | |
    | $\pm 0.1$ | |
    | $\pm 0.01$ | |
    | $\pm 0.001$ | |
    | $\pm 0.0001$ | |

    (b) From the table in (a), make a guess as to what $\lim_{x \to 0} f(x)$ should be. Write your guess below. (Try to figure out what the actual fraction is, not the decimal approximation. This may take some intelligent guessing and testing using your calculator.)

(c) It turns out that your guess in (b) is the right answer (assuming that you answered the question correctly). However, now try to see what happens at $x = \pm 0.00000001$ (that's seven zeros and then a 1). What do you get?

(d) This seems bizarre. We plugged in a value that was a lot closer to $x = 0$ than any of the other values that we plugged in from our table in part (a). What should we have expected to happen?

Hmmm, what does this mean? Does it mean that our guess above was wrong? (These are rhetorical questions and you do not need to answer them.) The problem is that our calculator just lied to us in the last calculation; the value it returned to you is wrong (unless you have a really fancy calculator that I don't know about).

The moral of the story is that just plugging in values is not sufficient for determining limits. Our calculator may lie! We'll come up with techniques for finding limits like this later.

2. Let $g(x) = \sin \dfrac{\pi}{x}$. Now, we would like to find $\lim\limits_{x \to 0} g(x)$. Again, it looks like something interesting may be happening at $x = 0$.

   (a) Let's take a look at what happens at some $x$-values that are pretty close to 0. Using your calculator, complete the following table.

   | $x$ | $g(x)$ |
   | --- | --- |
   | $\pm 0.1$ | |
   | $\pm 0.01$ | |
   | $\pm 0.001$ | |
   | $\pm 0.0001$ | |

   (b) What's your guess for the limit?

   (c) Now, complete this next table using your calculator. (Round your answers to 5 decimal places.)

   | $x$ | $g(x)$ |
   | --- | --- |
   | $\pm 0.3$ | |
   | $\pm 0.03$ | |
   | $\pm 0.003$ | |
   | $\pm 0.0003$ | |

   (d) OK, now what can you conclude?

   There is a picture of the graph of this function on page 69 of our book. Again, we can conclude that it is *not* sufficient to just plug in a few values "near" the $x$-value in question to deduce the limit.