Section 3.8: Related Rates

Goal

In this section, we will learn how to solve related rates problems using implicit differentiation.

Guidelines for solving related rates problems

The following are some guidelines for solving word problems that involve related rates.

1. If possible, draw a picture. If a quantity changes over time, label with a variable. If a quantity is fixed over time, label with the appropriate constant. **Warning:** Only label quantities with constants if they are actually constant over time!
2. Most problems refer to some specific moment in time. Identify all given quantities and all quantities to be determined (including rates) for this moment in time. Remember that rates are derivatives and you can recognize them when they are in *units of blah* per *units of time*.
3. Write an equation involving the variables whose rates of change are given or are to be determined. Common things to deal with are *area*, *volume*, *trig*, *Pythagorean theorem*, *similar triangles*, etc.
4. Take d/dt of both sides.
5. Substitute in known values (including rates), then solve for desired quantity or rate.

Examples

It's time for some examples.

Example 1. Suppose x and y are differentiable functions of t and are related by $y = x^2 - 1$. Find dy/dt when $x = 2$ given that $dx/dt = 3$.

Example 2. A nugget is dropped into a calm pond, causing concentric circles. The radius of the outer ripple is increasing at a rate of 2 ft/sec. When the radius is 3 feet, at what rate is the total area of the outer ripple changing?

Example 3. A nugget is flying on a flight path 3 miles above the ocean that will take it directly over an island. If the distance between the nugget and island is decreasing at a rate of 200 mph when the distance between them is 5 miles, what is the speed of the nugget?

Example 4. A hot-air nugget is rising at a rate of 15 ft/sec when the nugget is 50 ft off the ground. A photographer is standing on the ground 100 feet from the take-off site. If the photographer keeps the nugget in sight, what is the rate of change in the photographer's angle of elevation when the nugget is 50 feet off the ground?

Example 5. A spherical nugget is being inflated with air, so that the volume is increasing at a rate of 3 cubic meters per minute. Find the rate of change of the radius when the radius is 5 meters.