

Section 4.5: Summary of Curve Sketching (part 2)

Curvilinear Asymptotes

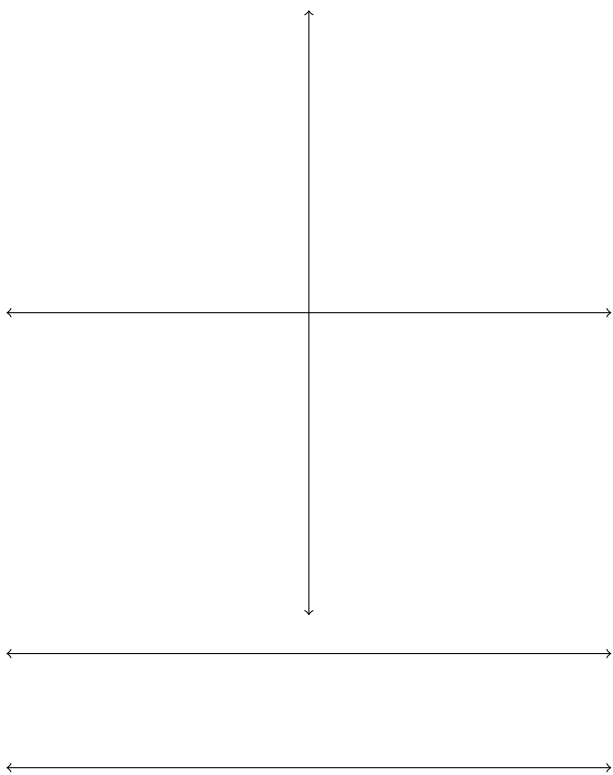
Question 1. If $f(x) = p(x)/q(x)$ is a rational function such that $p(x)$ and $q(x)$ have no factors in common (i.e., the “fraction” is reduced), then when will $f(x)$ have a horizontal asymptote? When will it not?

Answer 2.

When the degree of the numerator is _____ than the degree of the denominator, other kinds of asymptotes are possible: *curvilinear* (sometimes called *slant* or *oblique* if degree is 1). To see what these new kinds of asymptotes are, we use polynomial long division.

Theorem 3. *A rational function cannot have both a horizontal asymptote and a curvilinear (including slant) asymptote. Why?*

Example 4. Identify the curvilinear asymptote of $g(x) = \frac{x^3}{x^2 + 1}$ and sketch its graph.



Example 5. Sketch the graph of $f(x) = \frac{x^2 + x + 1}{x - 1}$.

