MA 2550: Calculus I (Fall 2009)
Review for Exam 1

Exam 1 covers material in sections 2.2, 2.3, 2.5, and most of 3.1, as well as the material covered in the Diagnostics Tests. This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; you may do what you want with it.

Topics

To be successful on Exam 1 you should

- understand what a function is and be able to determine whether a given graph/equation represents a function (know vertical line test)
- know definitions of the six trigonometric functions, basic trig identities, and be able to evaluate trig functions at the standard angles on the unit circle
- know shapes of basic graphs (including trig functions)
- understand transformations of graphs and be able to sketch transformations of a given graph
- be able to find domain and range
- be able to find combinations of functions (sum, difference, product, quotient, and composition)
- know formulas for slope and equations of lines
- be able to find equation of a line passing through two points
- know intuitive definition of limit (using the word "near")
- be able to come up with examples of graphs AND equations of functions that illustrate various possibilities involving limits (for example, be able to provide an example of an equation such that both the limit from the left and the limit from the right exist, but are not equal)
- know limit laws (including Squeeze Theorem)
- be able to evaluate limits (including problems that require Squeeze Theorem)
- know definition of continuity and have an intuitive understanding of continuity
- know statement of Intermediate Value Theorem and be able to use it to solve problems similar to homework
- know limit definition of the derivative at a point and be able to find derivatives using this definition
- be able to find the slope of the tangent line to the graph of a given function at a specified point
- be able to find the equation of the tangent line to a given graph at a specified point

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points. On the other hand, if you have the wrong answer because of a silly computational mistake, but have shown that you have an understanding of the material being tested, then you will receive most of the points.
- I will be grading the justification of your answer, not just the answer. So, you must use proper notation and make appropriate conclusions.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits where they are needed! This goes along with using proper notation and making appropriate conclusions.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that this concept is somehow more important than ones that do not appear frequently.

Note that a typo on Exercise 14 has been fixed.

1. True or False? Justify your answer.

   (a) The functions $u(x) = \dfrac{\sqrt{x}(x+1)}{x^2+x}$ and $v(x) = \dfrac{1}{\sqrt{x}}$ are equal.

(b) $f \circ g = g \circ f$ holds for arbitrary functions f and g.

(c) If a function $f(x)$ does not have a limit as x approaches a from the left, then $f(x)$ does not have a limit as x approaches a from the right.

(d) If a function is not continuous at $x = a$, then either it is not defined at $x = a$ or it does not have a limit as x approaches a.

(e) If f is a function such that $f(0) < 0$ and $f(2) > 0$, then there is a number c in the interval $(0, 2)$ such that $f(c) = 0$.

(f) If $h(x) \leq f(x) \leq g(x)$ for all real numbers x and $\lim_{x \to a} h(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a} f(x)$ also exists.

(g) Exercise 5 in True-False Quiz, page 108.

(h) Exercise 9 in True-False Quiz, page 108.

2. Determine whether each of the following rules define y as a function of x. If not, provide a reason why.

   (a) $x^2 + 2x + y^3 = 17$

   (b) $x = \sin(y)$

3. Determine whether each of the following scenarios could define a function. If so, identify the domain and range. If not, explain why.

   (a) A set of 10 missiles get launched and each missile follows a set of directions that tell the missile what target to blow up. Some of the missiles land on the same target.

   (b) A person types a phrase into the search engine Google, hits enter, and multiple entries are returned.

4. A camera is mounted at a point on the ground 500 meters from the base of a space shuttle launching pad. The shuttle rises vertically when launched, and the camera's angle of elevation is continually adjusted to follow the bottom of the shuttle. Express the height x as function of the angle of elevation θ.

5. Sketch the graph of each function without using a calculator. In each case, identify how you obtained the graph from the graph of the function in parentheses.

   (a) $f(x) = -1 + \sqrt{2 - x}$ $\quad (y = \sqrt{x})$

   (b) $g(x) = \dfrac{2x - 1}{x}$ $\quad (y = \dfrac{1}{x}$; Hint: first, split into two fractions.)

6. The graph of $y = a\dfrac{1}{\cos(x + b) + 2} + c$ results when the graph of $y = \dfrac{1}{\cos(x) + 2}$ is reflected over the x-axis, shifted 3 units to the right, and then shifted 4 units down. Find a, b, and c.

7. If $f(x) = 8 - x$ and $g(x) = 3x^2 - x + 4$, find formulas for $(g \circ f)(x)$ and $(f \circ g)(x)$.

8. Sketch the graph of a possible function f that has all properties (a)–(g) listed below.

   (a) The domain of f is $[-1, 2]$

   (b) $f(0) = f(2) = 0$

   (c) $f(-1) = 1$

(d) $\lim_{x\to 0^-} f(x) = 0$

(e) $\lim_{x\to 0^+} f(x) = 2$

(f) $\lim_{x\to 2^-} f(x) = 1$

(g) $\lim_{x\to -1^+} f(x) = -1$

9. Sketch the graphs of possible functions f, g, and h such that: f satisfies property (a) below, g satisfies property (b) below, and h satisfies property (c) below. (There should be three separate graphs.)

   (a) $\lim_{x\to 0} f(x) = f(0)$

   (b) $\lim_{x\to 0^-} g(x) = -1$ and $\lim_{x\to 0^+} g(x) = +1$

   (c) $\lim_{x\to 0} h(x) \neq h(0)$, where $h(0)$ is defined.

10. Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE).

   (a) $\lim_{x\to 2} x^2 + 4x - 12$

   (b) $\lim_{x\to 2} \dfrac{x^2 + 4x - 12}{x^2 + 4x + 3}$

   (c) $\lim_{x\to 2} \dfrac{x^2 + 4x - 12}{x^2 - x - 2}$

   (d) $\lim_{x\to 2} \dfrac{x^2 + 4x - 12}{x^2 - 4x + 4}$

   (e) $\lim_{x\to -3} \dfrac{x}{x+3}$

   (f) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x} - 2}$

   (g) $\lim_{x\to 3} \dfrac{1}{x-3}$

   (h) $\lim_{x\to 3} \dfrac{1}{(x-3)^2}$

   (i) $\lim_{x\to \pi} \dfrac{x}{\cos x}$

   (j) $\lim_{x\to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x}$

   (k) $\lim_{x\to 5} \dfrac{|x-5|}{x-5}$

11. Let f be defined as follows.

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ 3x + 4 & \text{if } 0 \leq x \leq 4 \\ x^2 & \text{if } x > 4 \end{cases}$$

   For (a)–(h), evaluate the limits if they exist. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). For part (i), answer the question.

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $f(0)$

(e) $\lim_{x \to 4^+} f(x)$

(f) $\lim_{x \to 4^-} f(x)$

(g) $\lim_{x \to 4} f(x)$

(h) $f(4)$

(i) Determine where f is continuous.

12. Let f and g be functions such that $\lim_{x \to a} f(x) = -3$ and $\lim_{x \to a} g(x) = 6$. Evaluate the following limits, if they exist.

    (a) $\lim_{x \to a} \frac{(g(x))^2}{f(x) + 5}$

    (b) $\lim_{x \to a} \frac{7f(x)}{2f(x) + g(x)}$

    (c) $\lim_{x \to a} \sqrt[3]{g(x) + 2}$

13. If $3x \leq f(x) \leq x^3 + 2$ for $0 \leq x \leq 2$, evaluate $\lim_{x \to 1} f(x)$.

14. Use the Squeeze Theorem to prove that $\lim_{x \to 0} x^4 \cos(2/x) = 0$.

15. Exercise 40, page 107.

16. Exercise 41, page 107.

17. Exercise 47, page 107.

18. Prove that the following function is *not* continuous on the interval $[0, 1]$. (Hint: Use the Intermediate Value Theorem.)
$$f(x) = \frac{1}{x^5 + \pi x - 1}$$

19. Exercise 65, page 108.

20. Let $f(x) = x^2 - x$.

    (a) Find the slope of the tangent line at $x = 2$ using the limit definition of $m_{\tan}$.

    (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

21. Exercise 7, page 120.

22. Exercise 17, page 120.

23. Let $g(x) = |x|$. Using the limit definition, prove that $g'(0)$ does not exist. (You should show that the limit from the left and the limit from the right are not equal.)