MA 2550: Calculus I (Fall 2009)
Review for Exam 2

Exam 2 covers material in sections 3.1–3.6, 3.8, 4.1 and 4.2. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; you may do what you want with it.

Topics

To be successful on Exam 2 you should

- understand the various equivalences of average rate of change versus instantaneous rate of change
- be able to find the slope/equation of the tangent line to the graph of a given function at a specified point
- be able to sketch the graph of f' given the graph of f
- be able to provide an example of a function that is continuous everywhere, but not differentiable everywhere
- be able to find derivatives of functions using differentiation rules
- in particular, be able to apply the Power Rule, Product Rule, Quotient Rule, and Chain Rule appropriately
- know derivatives of $\sin x$, $\cos x$, $\tan x$, and $\sec x$
- know the limits involving sine and cosine that appear in the red boxes on pages 149 and 150, respectively
- understand implicit differentiation and be able to find derivatives of implicit functions
- be able to solve related rates word problems (you should be able to label your answers with the appropriate units)
- know definition of critical number and be able to find them
- understand the concept of local max/min
- understand the relationship between critical numbers and local max/min
- understand the concept of absolute max/min
- know the statement of and understand the Extreme Value Theorem
- be able to find absolute max/min of a continuous function on a closed interval
- know the statement of and understand the Mean Value Theorem (and Rolle's Theorem)
- be able to find the points guaranteed to exist according to the Mean Value Theorem
- be prepared to provide examples that illustrate various concepts (for example, you should be able to state an example of a function that has a critical number, but does not have a local max or local min)

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points. On the other hand, if you have the wrong answer because of a silly computational mistake, but have shown that you have an understanding of the material being tested, then you will receive most of the points.
- I will be grading the justification of your answer, not just the answer. So, you must use proper notation and make appropriate conclusions.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits where they are needed! This goes along with using proper notation and making appropriate conclusions.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.

   (a) $\dfrac{d}{dx}[f(cx)] = c\dfrac{d}{dx}[f(x)]$.

   (b) $\dfrac{d}{dx}\left[\dfrac{1}{f(x)}\right] = \dfrac{1}{f'(x)}$.

(c) $\dfrac{d}{dx}\left[(2x^3+5x^2-7)(3\cos(x)+13x)\right] = (6x^2+10x)(-3\sin(x)+13)$

(d) If $f'(c)=0$, then f has either a local minimum or local maximum at $x=c$.

(e) If f has a local minimum or local maximum at $x=c$, then $f'(c)=0$.

2. Let $f(x)=x^2-x$.

   (a) Find $f'(x)$ using the limit definition.

   (b) Find the equation of the tangent line to the graph of $f(x)$ at $x=2$.

3. Let $g(x)=|x|$. Using the limit definition, prove that $g'(0)$ does not exist. (You should show that the limit from the left and the limit from the right are not equal.)

4. Exercise 12, page 120

5. Exercise 13, page 120

6. Exercise 3, page 131

7. Exercise 4, page 132

8. Differentiate each of the following functions, but do *not* simplify.

   (a) $f(x)=\pi^2$

   (b) $g(x)=\dfrac{x}{3}+\sqrt{x}-\dfrac{1}{x}+\sqrt{2}$

   (c) $y=x^3\sqrt{x+1}$

   (d) $h(x)=\sin(\sin x)$

   (e) $k(x)=\sin^2 x$

   (f) $f(x)=\sin x^2$

   (g) $r(x)=\dfrac{x^2-3x+1}{2-x}$

   (h) $y=\dfrac{3x-2x^2}{x^{2/3}}$

   (i) $f(x)=2\cos x+\sec 2x$

   (j) $g(x)=\dfrac{1+\cos x}{1-\cos x}$

   (k) $y=\tan^3(1-5x^2)$

   (l) $y=x^2\sin\left(\dfrac{\pi}{4}x\right)$

9. If $f(1)=3$, $g(5)=2$, $h(1)=5$, $f'(2)=3$, $g'(5)=-2$, and $h'(1)=-8$, find $(f\circ g\circ h)'(1)$

10. Exercise 63(a) on page 162

11. For each of the following, find dy/dx.

    (a) $x^2y+y^2=x$

    (b) $x=\sin 2y$

12. Find the equation of the tangent line to the graph of $f(x)=2\sin x$ when $x=\pi/3$.

13. Find the equation of the tangent line to the graph of $g(x) = \sqrt{x-3}$ when $x = 7$.

14. Find the equation of the tangent line to the graph of $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

15. A spherical balloon is deflated so that its volume decreases at a constant rate of 3 in^3/sec. How fast is the balloon's diameter decreasing when the radius is 2 inches?

16. A 10-foot ladder is leaning against a building. If the top of the ladder slides down the wall at a constant rate of 2 feet per second, how fast is the acute angle the ladder makes with the ground decreasing when the top of the ladder is 5 feet from the ground? (Give the answer in radians per second.)

17. An interstellar nugget is circling the planet Earth a mile from its surface in a clockwise direction. An observer is standing at a fixed location watching the nugget with his nuggetscope. If the nugget is traveling at a rate of 200 mph, then what is the rate of change in the angle of elevation of the nuggetscope when the angle is $\pi/4$? For this problem, assume the surface of the Earth is locally flat (not round) and assume that the observer is standing facing the nugget with the nugget approaching.

18. Find all critical numbers of $f(x) = x\sqrt{4-x^2}$.

19. Exercise 11 on page 211

20. Find the absolute max and min for $f(x) = \cos x - x$ on the interval $[0, 2\pi]$.

21. Find the absolute max and min for $g(x) = 2x + \dfrac{1}{2x}$ on the interval $[1, 4]$.

22. Determine whether Rolle's Theorem applies to each of the following functions on the indicated interval. Explain your answer.

    (a) $f(x) = \dfrac{x^3 + x}{x}$, $[-1, 1]$

    (b) $g(x) = \dfrac{x^2}{x^2 - 3}$, $[-3/2, 3/2]$

23. Given $f(x) = 10 - \dfrac{16}{x}$, show that f satisfies the hypotheses of the Mean Value Theorem on the interval $[2, 8]$, and then find all numbers c that the Mean Value Theorem guarantee exist.

24. Exercise 33 on page 220

25. Provide an example of a function f such that $f'(2) = 0$, but f does not have a local maximum or local minimum at $x = 2$.