Introduction to Linear Algebra, Spring 2007
MATH 3130, Section 001
Review for Exam 1

Here is a review for your upcoming exam. The exam covers Sections 1.1-1.5, 1.7, and 1.8. Note that Sections 1.6 and 1.9 are not on the exam. This review should give you a good indication of what you are expected to know for the exam.

Most importantly, don't forget when the exam is:

Exam 1: Friday, February 9

For the exam, you will be expected to know the statements of definitions and theorems. In particular, you should know the difference between the definition of a particular concept and theorems about that concept. For example, a linear system is defined to be *consistent* if the system has at least one solution. But Theorem 2 (page 24) tells us that a system is consistent iff (if and only if) the rightmost column of the augmented matrix is not a pivot column. Here is a minimal list of the definitions and theorems that you must know:

Definitions:

- linear equation
- linear system
- solution set of a linear system
- consistent
- inconsistent
- coefficient matrix
- augmented matrix
- elementary row operations
- (row) echelon form (REF)
- reduced (row) echelon form (RREF)
- pivot and pivot column
- free variable
- (column) vector
- addition of vectors and scalar multiplication of vectors
- linear combination
- span
- matrix-vector product

- homogeneous system of equations
- trivial and nontrivial solutions
- parametric vector description of solution sets
- linear independence and dependence
- linear transformation

Theorems:

- Blue Box on page 4
- Theorem 2 (Existence and Uniqueness Theorem)
- Blue Box on page 30
- Blue Box on page 32
- Theorem 3
- Theorem 4
- Theorem 5
- Blue Box on page 50
- Theorem 6
- Theorem 7 (A special case is the Blue Box on page 67)
- Theorem 8
- Theorem 9
- Both Blue Boxes on page 77

Note that you are *not* required to memorize the proofs of any theorems. However, the proof of Theorem 7 contains some fundamental ideas and is worth understanding (not memorizing).

You should also be able to provide *simple* examples that satisfy definitions or theorems. For example, I would expect you to be able to provide me with an example of 3 vectors from $\mathbb{R}^3$ that are linearly independent (or dependent). You don't want to memorize examples, but rather the necessary characteristics that an example would need to have. In addition, you should be able to generate counterexamples to false statements. For example, I could ask you to provide a counterexample to the following (false) statement: Every collection of 3 vectors in $\mathbb{R}^3$ spans $\mathbb{R}^3$.

One of the fundamental concepts of this chapter is understanding what types of questions can be answered by tossing some numbers in a matrix and then row reducing the matrix into either echelon form or reduced echelon form. This is the essence of Theorem 3 and Theorem 4. You need to know when to create a matrix, when to row reduce a matrix, how to row reduce a matrix, and how to interpret a row reduced matrix.

Here are a few things that you want to keep straight.

- Don't confuse a coefficient matrix with an augmented matrix. For example, the A in $A\mathbf{x} = \mathbf{0}$ is a coefficient matrix of the corresponding system of linear equations. Also, some theorems involve statements about augmented matrices and some are about coefficient matrices. Sort out these differences.
- There are several theorems that involve statements about pivots. What types of questions can we answer by determining whether a matrix has a pivot in every *row*? And, what types of questions can we answer by determining whether a matrix has a pivot in every *column*?
- When do we need to row reduce a matrix to echelon form? When do we need to row reduce to reduced echelon form?
- If you are asked (on the exam or otherwise) to justify or prove that a statement is true, you need to *prove* the statement, which usually involves stating definitions, theorems, and possibly making a calculation. Providing an example of the statement "working" is not a proof. However, if you are asked to justify that a statement is *not* true, then providing a counterexample is exactly the kind of thing you want to do.

Here's the bottom line (no pun intended), the exam will be similar in nature to the homework, but you should not expect to see identical problems on exam. Lastly, be prepared to *justify* all of your answers and *show all of your work*.