Introduction to Linear Algebra, Spring 2007
MATH 3130, Section 001
Review for Exam 2

Here is a review for your upcoming exam. The exam covers Sections 1.9, 2.1-2.3, 2.5, 3.1-3.3, and 4.1. This review should give you a good indication of what you are expected to know for the exam. Also, this exam will be similar in nature to Exam 1, but perhaps more challenging.

Most importantly, don't forget when the exam is:

Exam 2: Friday, March 9

For the exam, you will be expected to know the statements of definitions and theorems. In particular, you should know the difference between the definition of a particular concept and theorems about that concept. For example, a square matrix A is defined to be *invertible* if there exists a matrix B such that $AB = BA = I$. But the Invertible Matrix Theorem (Theorem 2.8, page 129) tells us that there 11 other things that are equivalent to a matrix being invertible. Here is a minimal list of the definitions, theorems, and algorithms that you must know.

Definitions:

- linear transformation
- standard matrix for a linear transformation
- onto linear transformation
- one-to-one linear transformation
- definition of matrix multiplication (see page 110)
- transpose of a matrix
- elementary matrix (know all 3 types and what their determinants are)
- upper and lower triangular matrix
- LU factorization
- determinant of a square matrix
- cofactors and cofactor expansion
- vector space
- subspace

Theorems:

- Theorem 1.10
- Theorems 1.11 and 1.12
- Theorems 2.1, 2.2, and 2.3

- Theorem 2.5
- Theorem 2.6
- Theorem 2.7 and both Blue Boxes on page 123
- Invertible Matrix Theorem (Theorem 2.8)
- Blue Box on page 130
- Theorem 2.9
- Theorems 3.1, 3.2, and 3.3
- Theorem 3.4
- Theorems 3.5 and 3.6
- Cramer's Rule (Theorem 3.7)
- Theorem 3.9
- Theorem 3.10 and its Generalization on page 209
- Theorem 4.1

Note that you are *not* required to memorize the proofs of any theorems. Here is a list of some important things that you need to know how to do.

Things to Know and Algorithms:

- Find standard matrix for a linear transformation
- Determine whether a linear transformation is 1-1 or onto
- Find product of 2 matrices
- Determine if a matrix has an inverse
- Find inverse of a matrix (if it exists)
- Use the Invertible Matrix Theorem to answer questions about any of the equivalent conditions.
- Find an LU factorization and use it to solve a system of equations
- Find determinant of a matrix
- Find a solution to $A\mathbf{x} = \mathbf{b}$ for A invertible using Cramer's Rule
- Determine whether a set V is a vector space or not.

The most important Theorem that we have learned since the last test is the Invertible Matrix Theorem. Remember that if one of the equivalent conditions is true, then all of them are. Similarly, if one of the conditions is false, then all of the conditions are false. There are a couple of theorems that make improvements or additions to the IMT. For example, part (g) of the IMT says: The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$. But Theorem 2.5, tells us that if A is invertible, then $A\mathbf{x} = \mathbf{b}$ has a *unique* solution (namely, $\mathbf{x} = A^{-1}\mathbf{b}$). Also, Theorem 3.4 says that a square matrix A is invertible iff $\det A \neq 0$. So, you can add that to the IMT.

You should also be able to provide *simple* examples that satisfy definitions or theorems. For example, I would expect you to be able to provide me with an example of a vector space that is not *equal* to some $\mathbb{R}^n$. You don't want to memorize examples, but rather the necessary characteristics that an example would need to have. In addition, you should be able to generate counterexamples to false statements. For example, I could ask you to provide a counterexample to the following (false) statement: Every square matrix is invertible.

Here are a few things that you want to keep straight.

- As with the material on Exam 1, there are several questions that we can answer by determining whether a matrix has a pivot in every row or column.
- If you are asked (on the exam or otherwise) to justify or prove that a statement is true, you need to *prove* the statement, which usually involves stating definitions, theorems, and possibly making a calculation. Providing an example of the statement "working" is not a proof. However, if you are asked to justify that a statement is *not* true, then providing a counterexample is exactly the kind of thing you want to do.

Here's the bottom line, the exam will be similar in nature to the homework, but you should not expect to see identical problems on exam. Lastly, be prepared to *justify* all of your answers and *show all of your work*.