Introduction to Linear Algebra, Spring 2007
MATH 3130, Section 001
Review for Exam 3

Here is a review for your upcoming exam. The exam covers Sections 4.2-4.6, 5.1-5.3, and 6.1. This review should give you a good indication of what you are expected to know for the exam. Also, this exam will be similar in nature to the last 2 exams.

Most importantly, don't forget when the exam is:

Exam 3: Friday, April 13

For the exam, you will be expected to know the statements of definitions and theorems. In particular, you should know the difference between the definition of a particular concept and theorems about that concept. Here is a minimal list of the definitions, theorems, and algorithms that you must know. Note that you are *not* required to memorize the proofs of any theorems.

Definitions:

- null space of a matrix
- column space of a matrix
- kernel of a linear transformation
- range of a linear transformation
- linear independence and dependence for vectors of an arbitrary vector space
- basis of a vector space
- standard basis for $\mathbb{R}^n$
- standard basis for $\mathbb{P}_n$
- coordinates of a vector relative to a given basis
- coordinate vector of a vector relative to a given basis
- change-of-coordinate matrix $\mathcal{P}_\mathcal{B}$
- dimension of a vector space
- row space of a matrix
- rank and nullity of a matrix
- eigen vector, eigen value, and eigen space
- characteristic equation
- similar matrices
- diagonalizable
- length (or norm) of a vector

- distance between two vectors
- orthogonal
- orthogonal complement

Theorems:

- Theorems 4.2 and 4.3
- Blue Box on page 230
- Theorems 4.4
- Theorem 4.5 (The Spanning Set Theorem)
- Theorem 4.6
- Theorem 4.7 (The Unique Representation Theorem)
- Theorem 4.8
- Theorems 4.9, 4.10, 4.11
- Theorem 4.12 (The Basis Theorem)
- Blue Box on page 260
- Theorem 4.13
- Theorem 4.14 (The Rank Theorem)
- Invertible Matrix Theorem (All parts; pages 129, 267, 312
- Theorem 5.1 and 5.2
- Blue Box on page 313 (below Theorem 5.3)
- Theorem 5.4
- Theorem 5.5 (The Diagonalization Theorem)
- Theorems 5.6 and 5.7
- Theorem 6.1 and Blue Box on page 377
- Theorem 6.2
- Blue Box on page 380
- Theorem 6.3
- Blue Box on page 381

Algorithms and Things to Know:

- Given a matrix, find dimension and a basis for null space

- Given a matrix, find dimension and a basis for column space
- Given a matrix, find dimension and a basis for row space
- Determine whether a given collection of vectors in an arbitrary vector space is linearly independent or a spanning set
- Find eigen values for a matrix and their corresponding eigen vectors, as well as a basis for corresponding eigen space
- Given a matrix A, find P and D that diagonalize A
- Given a subspace W and a vector $\mathbf{v}$, determine whether $\mathbf{v} \in W^{\perp}$

The most important Theorem that we have learned this semester is the Invertible Matrix Theorem. Remember that if one of the equivalent conditions is true, then all of them are. Similarly, if one of the conditions is false, then all of the conditions are false. You should know all parts of the IMT. Also, don't forget that it only applies to square matrices.

You should also be able to provide *simple* examples that satisfy definitions or theorems. For example, I would expect you to be able to provide me with an example of a matrix with only a single eigen value. You don't want to memorize examples, but rather the necessary characteristics that an example would need to have. In addition, you should be able to generate counterexamples to false statements. For example, I could ask you to provide a counterexample to the following (false) statement: Every square matrix is diagonalizable.

Here are a few things that you want to keep straight.

- As with the material on the last two exams, there are several questions that we can answer by determining whether a matrix has a pivot in every row or column. This is exactly the kind of thing you want to be thinking about when answering the matching questions on the first page of all the exams.
- If you are asked (on the exam or otherwise) to justify or prove that a statement is true, you need to *prove* the statement, which usually involves stating definitions, theorems, and possibly making a calculation. Providing an example of the statement "working" is not a proof. However, if you are asked to justify that a statement is *not* true, then providing a counterexample is exactly the kind of thing you want to do.

Here's the bottom line, the exam will be similar in nature to the homework, but you should not expect to see identical problems on exam. Lastly, be prepared to *justify* all of your answers and *show all of your work*.