MATH 1300: Calculus I, Spring 2008
MIDTERM EXAM 3

April 9, 2008

YOUR NAME:

001	N. Flores	(8am)
002	A. Angel	(9am)
003	D. Ernst	(9am)
004	M. Formichella	(10am)
005	I. Becker	(11am)
006	D. Vernerey	(11am)
007	J. Harper	(12pm)
008	L. Harris	(2pm)
009	R. Krieger	(2pm)
011	R. Grover	(10am)
012	I. Mishev	(12pm)
013	R. Chestnut	(1pm)
014	I. Becker	(1pm)
015	D. McCarl	(3pm)
017	N. Flores	(10am)

After you get the test back, if you consider that something was incorrectly graded,
DO NOT WRITE ON YOUR TEST!
As clearly as possible write down your version of the story on a clean sheet of paper, attach it to your test, and give it back to your instructor for further consideration.

problem	points	score
1	10 pts	
2	10 pts	
3	6 pts	
4	12 pts	
5	25 pts	
6	15 pts	
7	8 pts	
8	8 pts	
9	6 pts	
TOTAL	100 pts	

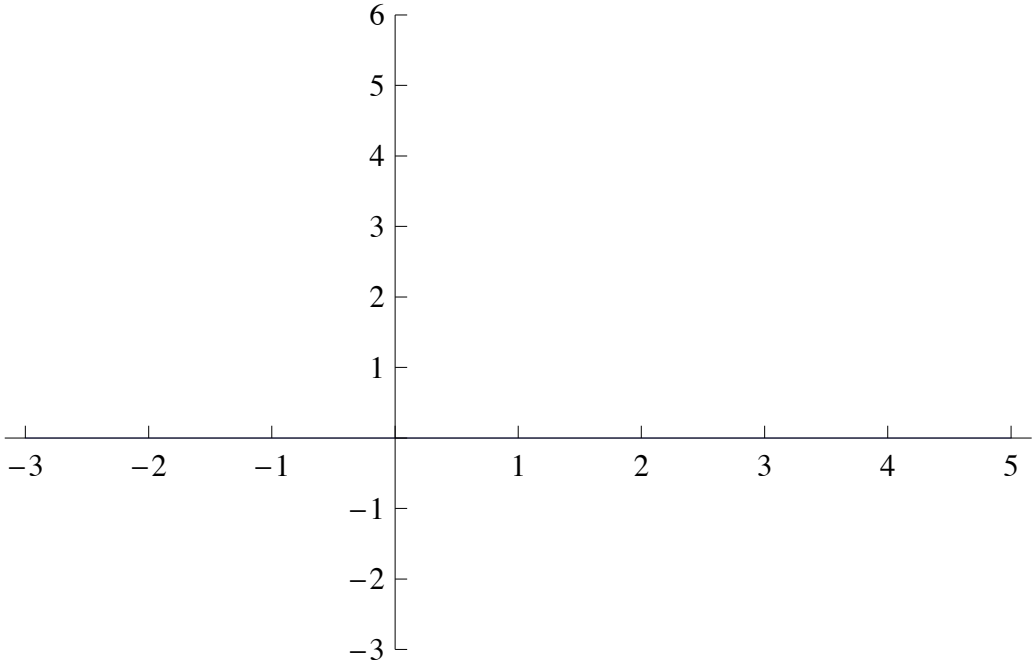
"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

INSTRUCTIONS: Answer each of the following questions. In order to receive full credit, your answers must be **complete**, **legible**, and **correct**. You must also **show all of your work** and give adequate explanations where necessary. No calculators, no books, no notes are allowed on this exam.

1. (10 pts) Find the absolute extrema of $f(x) = x^3 - x^2 - x + 2$ on the interval $[-1, 2]$.

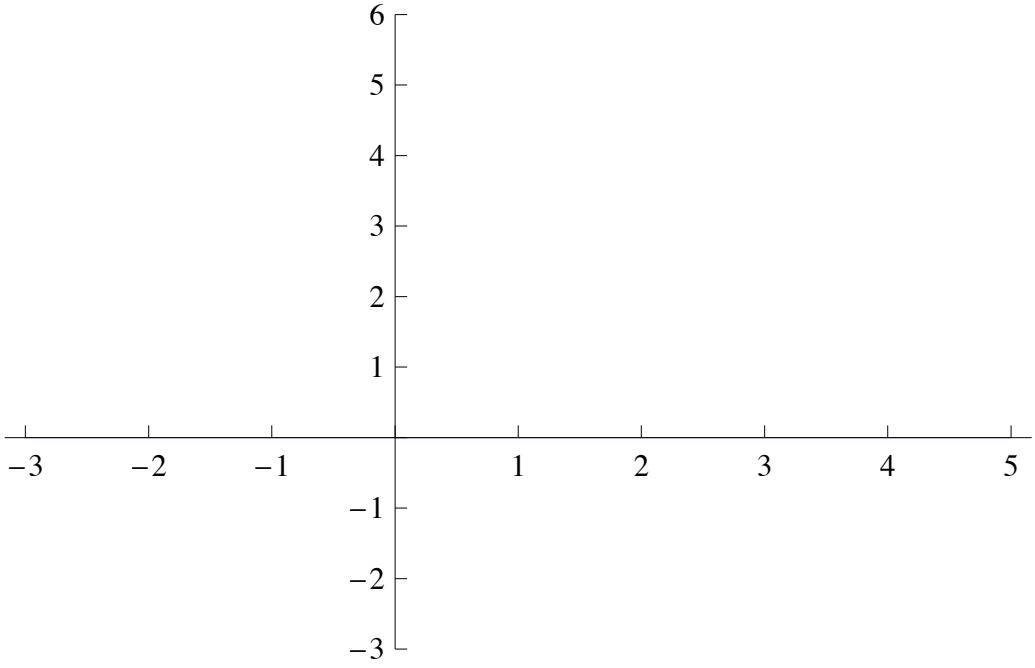
2. (a) (5 pts) Sketch the graph of a function $g(x)$ on the axes below, where $g(x)$ is continuous on $(-\infty, \infty)$, $g(x)$ is decreasing everywhere, and $g(x)$ is concave up everywhere.



(b) (5 pts) Sketch the graph of $f(x)$ on the axes below, where $f(x)$ is a continuous function on $(-\infty, \infty)$ that satisfies the following conditions:

x	$-\infty < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x < \infty$
$f(x)$	positive	5	positive	1	positive	3	positive
$f'(x)$	positive	DNE	negative	0	positive	2	positive
$f''(x)$	positive	DNE	positive	2	positive	0	negative

Hint: First, plot the points corresponding to $x = -1, 1$, and 2.

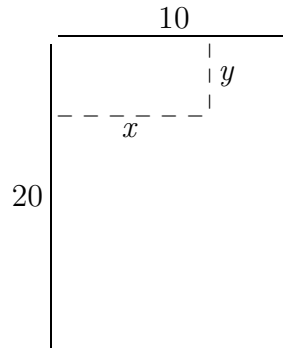


3. (6 pts) The function

$$f(x) = 10 - \frac{16}{x}$$

satisfies the hypotheses of the Mean Value Theorem over the interval $[2, 8]$. Find the value(s) of $x = c$ whose existence is guaranteed by the Mean Value Theorem.

4. Suppose you want to build a enclosed, rectangular pen for your cute German Shepherd puppy. For two sides of the pen you are going to use two perpendicular stone walls in your backyard, whose total lengths are 10 ft and 20 ft, respectively, and for the other two sides you are going to use 24 ft of fencing (See drawing).



(a) (4 pts) Write an equation for the enclosed area of the pen as a function of x.

(b) (2 pts) Determine the domain of the above area function.

(c) (6 pts) Find the dimensions of the pen that has maximum area.

5. (5 pts each) Evaluate each of the following indefinite integrals.

(a) $\int x(4+x^3)\,dx$

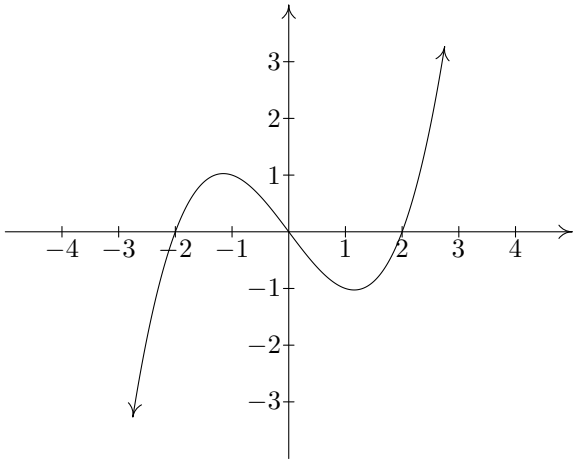
(b) $\int \frac{4+2x^{3/2}}{\sqrt{x}}\,dx$ (Hint: simplify first)

(c) $\int \frac{\cos^3 x}{1-\sin^2 x}\,dx$

(d) $\int \frac{\ln(x)}{x}\,dx$

(e) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

6. (3 pts each) Let $f(x)$ be continuous over $(-\infty, \infty)$. The following is the graph of the ***derivative*** of $f(x)$:



Graph of f'

(a) Find the x-coordinates of all points on the graph of $f(x)$ where the tangent line is horizontal.

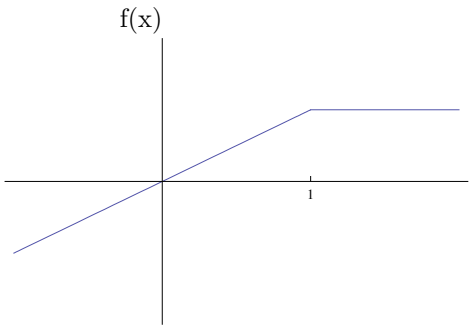
(b) Find the intervals, if any, on which $f(x)$ is increasing and decreasing.

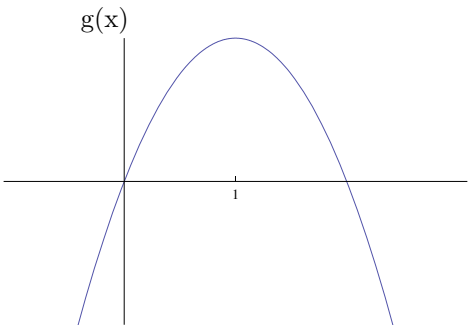
(c) Find the x-coordinates of all the relative maxima of $f(x)$.

(d) Find the x-coordinates of all the relative minima of $f(x)$.

(e) Find the approximate x-coordinates of all of the inflection points of $f(x)$.

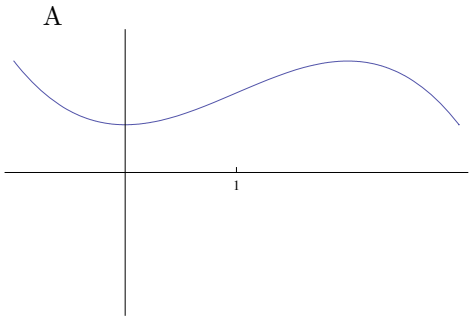
7. (4 pts each) Fill in the blanks corresponding to the graph of each function f and g with the appropriate letter below.

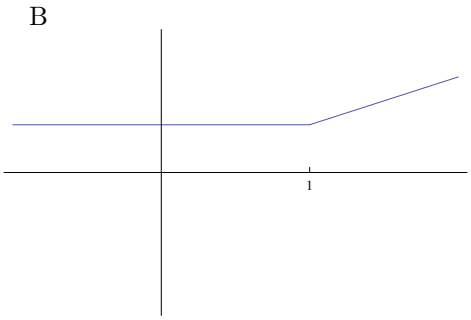


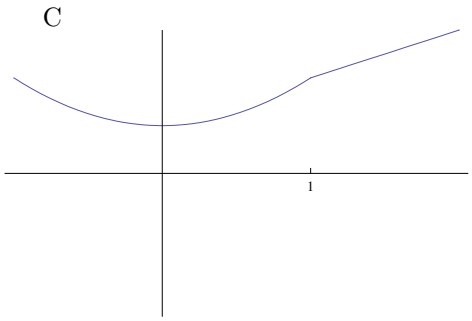


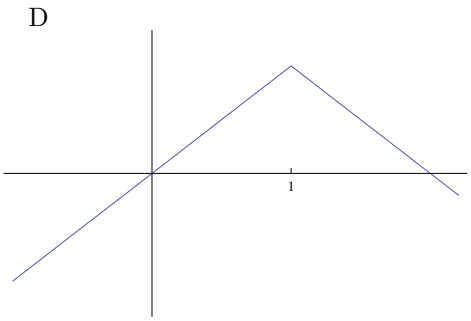
(a) A possible antiderivative of $f(x)$ is _____.

(b) A possible antiderivative of $g(x)$ is _____.









8. (8 pts) Find *all* aymptotes of the function

$$f(x) = \frac{2x^3 - 2x^2 - 1}{x - 1}.$$

9. Let f be a function that has an inverse, denoted by f^{-1}. Use facts about inverse functions to answer the following questions.

   (a) (3 pts) Suppose that $f(2) = 2$ and $f(4) = 6$. Find the equation of the secant line (also called chord) to the graph of f^{-1} through the pair of points whose x-coordinates are $x = 2$ and $x = 6$.

   (b) (3 pts) Explain why the graph of f and the graph of f^{-1} are symmetric about the line $y = x$.