MATH 1300: CALCULUS 1

December 14, 2005

FINAL EXAM

YOUR NAME:

- ◯ **001** A. Pence .... (8am)
- ◯ **002** A. Spina .... (8am)
- ◯ **003** I. Becker .... (8am)
- ◯ **004** E. Frugoni .... (8am)
- ◯ **005** T. Seguin .... (9am)
- ◯ **006** I. Mishev .... (9am)
- ◯ **007** J. Johanson .... (9am)
- ◯ **008** J. Sanders .... (9am)
- ◯ **009** J. Nibert .... (9am)
- ◯ **010** J. Wiscons .... (10am)
- ◯ **012** V. Wong .... (11am)
- ◯ **013** E. Kim .... (11am)
- ◯ **014** T. Schumacher .... (12pm)
- ◯ **015** J. Clelland .... (1pm)
- ◯ **016** J. Meadows .... (2pm)
- ◯ **018** E. Angel .... (4pm)
- ◯ **019** C. Seacrest .... (4pm)

Show all your work.
Answers out of the blue and without any supporting work will receive no credit *even if they are right!*
(Does not apply to multiple choice and true/false questions)

Write clearly. **Box your final answers.**
No calculators allowed. **No cheat sheets allowed.**

DO NOT WRITE ON THIS BOX!

problem	points	score
1	25 pts	
2	20 pts	
3	7 pts	
4	15 pts	
5	13 pts	
6	20 pts	
7-26	100 pts	
TOTAL	200 pts	

1: *(25 points)* Evaluate the following definite and indefinite integrals:

(a) $\int \left(x^2 + \frac{2}{x^{5/2}} \right) dx$

(b) $\int (3e^{5x} + 7)\, dx$

(c) $\int_0^{\pi/2} e^{1-\sin x} \cos x \, dx$

(d) $\int \cot x \, dx$

(e) $\int_e^{e^3} \frac{1}{x(\ln x)^2}\, dx$

2: *(20 points)* Find the area under the curve $y = x$ over the interval $[1, 2]$ using each of the following methods. (Please note that we will be grading your PROCESS more than your answers on this question, so you MUST SHOW APPROPRIATE STEPS to receive any credit.)

(a) Sketch the curve and use a formula from geometry.

(b) Use Riemann sums with right-hand endpoints (i.e., $x_k^* = x_k = a + k\Delta x$). You may find the follwing formulae useful:

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

(c) Express the area as a definite integral and evaluate using the Fundamental Theorem of Calculus.

3: *(7 points)* Evaluate the integral $\int_2^4 (5f(x) - \frac{1}{3}g(x))\,dx$, given that

$$\int_2^4 f(x)\,dx = \tfrac{1}{4}, \qquad \int_2^4 g(x)\,dx = 5.$$

4: *(15 points)* Define $F(x)$ by

$$F(x) = \int_{\pi/2}^{x} \cos t\,dt.$$

(a) Use Part 2 of the Fundamental Theorem of Calculus to find $F'(x)$.

(b) Check the result in part (a) by first integrating and then differentiating.

5: *(13 points)* Find the area of the region enclosed by the curves $y = x^2$ and $y = 3x$.

6: *(20 points)* Set up, but DO NOT EVALUATE, integrals that express the volume of the solids that result when the region enclosed by the curves

$$y = 0, \quad y = \sin x, \quad x = 0, \quad x = \pi$$

is revolved about:

(a) the x-axis

(b) the y-axis

The remainder of this exam consists of **True/False** and **Multiple Choice** questions. Please circle your answer for each question on **THIS** page. (Questions are on the pages that follow.) **It is not necessary to show your work, and no partial credit will be given.**

(5 points each)

7:	(A)	(B)	(C)	(D)	(E)
8:	(A)	(B)	(C)	(D)	(E)
9:	(A)	(B)	(C)	(D)	(E)
10:	(TRUE)		(FALSE)		
11:	(TRUE)		(FALSE)		
12:	(A)	(B)	(C)	(D)	(E)
13:	(A)	(B)	(C)	(D)	(E)
14:	(A)	(B)	(C)	(D)	(E)
15:	(A)	(B)	(C)	(D)	(E)
16:	(TRUE)		(FALSE)		
17:	(A)	(B)	(C)	(D)	(E)
18:	(A)	(B)	(C)	(D)	(E)
19:	(A)	(B)	(C)	(D)	(E)
20:	(A)	(B)	(C)	(D)	(E)
21:	(A)	(B)	(C)	(D)	(E)
22:	(TRUE)		(FALSE)		
23:	(A)	(B)	(C)	(D)	(E)
24:	(A)	(B)	(C)	(D)	(E)
25:	(A)	(B)	(C)	(D)	(E)
26:	(TRUE)		(FALSE)		

7: The graph of the function $y = \dfrac{10x+3}{2x-5}$ has a horizontal asymptote at:

(A) $y = 5$ **(B)** $y = \dfrac{3}{5}$ **(C)** $y = -\dfrac{3}{5}$ **(D)** $y = -5$ **(E)** $y = 0$

8: $\lim_{x \to 6} 7 =$

(A) 1 **(B)** -1 **(C)** 7 **(D)** Does not exist **(E)** -7

9: $\lim_{x \to 3} \dfrac{x}{x-3} =$

(A) 1 **(B)** 0 **(C)** $+\infty$ **(D)** Does not exist **(E)** $-\infty$

10: TRUE or **FALSE**: The equation $f(x) = x^3 + x - 3 = 0$ has at least one solution on the interval $[1, 2]$.

11: TRUE or **FALSE**: $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin x} = 0$

12: Find the equation of the tangent line to the curve $y = 2x$ at $x = 3$.

(A) $y = 2x$ **(B)** $y = 2x - 3$ **(C)** $y = 2x + 3$ **(D)** $y = 2$ **(E)** $y = 3$

13: Find $\frac{dy}{dx}$ if $y = e^8$.

(A) $7e^7$ **(B)** $8e^7$ **(C)** -8 **(D)** 8 **(E)** 0

14: Suppose that $g(x) = \sqrt{x} f(x)$. Find $g'(1)$, given that $f(1) = 8$ and $f'(1) = 5$.

(A) 5 **(B)** 4 **(C)** 9 **(D)** 13 **(E)** 0

15: Find $f'(x)$ if $f(x) = x^3 \cos x$.

(A) $3x^2 \cos x$ **(B)** $-3x^2 \cos x$ **(C)** $3x^2 \cos x + x^3 \sin x$

(D) $3x^2 \cos x - x^3 \sin x$ **(E)** $3x^2 \sin x$

16: TRUE or **FALSE:** $\dfrac{d^{71}}{dx^{71}}(\sin x) = \cos x$.

17: Find $\dfrac{dV}{dt}$ for a spherical balloon of radius 2 ft if $\dfrac{dr}{dt} = 0.5\dfrac{\text{ft}}{\text{s}}$. (Recall that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.)

(A) $\dfrac{16\pi}{3}\dfrac{\text{ft}^3}{\text{s}}$ **(B)** $\dfrac{32\pi}{3}\dfrac{\text{ft}^3}{\text{s}}$ **(C)** $8\pi\dfrac{\text{ft}^3}{\text{s}}$ **(D)** $4\pi\dfrac{\text{ft}^3}{\text{s}}$ **(E)** $2\pi\dfrac{\text{ft}^3}{\text{s}}$

18: Find $\dfrac{dy}{dx}$ if $y = \ln(4x^2)$.

(A) $\dfrac{1}{x}$ **(B)** $\dfrac{2}{x^2}$ **(C)** $\dfrac{1}{2x^2}$ **(D)** $\dfrac{1}{x^2}$ **(E)** $\dfrac{2}{x}$

19: Find $\dfrac{dy}{dx}$ if $x^3 + 3y^2 = 9$.

(A) $\dfrac{9-3x^2}{6y}$ **(B)** $-\dfrac{x^2}{2y}$ **(C)** $\dfrac{x^2}{2y}$ **(D)** $\dfrac{9+3x^2}{6y}$ **(E)** $3x^2$

20: $\lim_{x \to +\infty} \frac{\ln x}{e^x} =$

(A) 0 **(B)** $+\infty$ **(C)** $-\infty$ **(D)** 1 **(E)** -1

21: The largest interval on which $f(x) = x^2 + 4x + 2$ is increasing is

(A) $[0, +\infty)$ **(B)** $(-\infty, 0]$ **(C)** $[-2, +\infty)$ **(D)** $(-\infty, -2]$ **(E)** $(-\infty, +\infty)$

22: TRUE or **FALSE**: The function $f(x) = \sqrt{x-6}$ is concave down on its entire domain.

23: Where is the function $f(x) = \cos x$ increasing on the interval $[0, 2\pi]$?

(A) $[0, \pi]$ **(B)** $[\pi, 2\pi]$ **(C)** $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(D) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$ **(E)** $[0, 2\pi]$

24: The weekly profit function for a certain company is

$$P(x) = -\tfrac{1}{10}x^2 + 30x - 500$$

where x is the number of the company's product that is made and sold. How many individual items of the product must the company make and sell weekly in order to maximize its profit?

(A) 300 **(B)** 50 **(C)** 500 **(D)** 150 **(E)** 200

25: The function $f(x) = \dfrac{1}{x}$ has an absolute maximum on the interval $[1, 3]$ of

(A) 1 **(B)** $\dfrac{1}{9}$ **(C)** 9

(D) $\dfrac{1}{3}$ **(E)** No absolute maximum exists

26: **TRUE** or **FALSE**: The hypotheses of the Mean Value Theorem are satisfied for the function $f(x) = \dfrac{1}{x^8} - 1$ on the interval $[-1, 1]$.