MATH 1300: CALCULUS 1

May 9, 2007

FINAL EXAM

YOUR NAME:

Fill in the bubble that corresponds to your section

- ◯ **001** P. NEWBERRY ..... (8AM)
- ◯ **002** M. HEDGES ..... (9AM)
- ◯ **003** J. KISH ..... (9AM)
- ◯ **004** R. CHESTNUT ..... (10AM)
- ◯ **005** C. MESA ..... (11AM)
- ◯ **006** M. STACKPOLE ..... (11AM)
- ◯ **007** I. BECKER ..... (12PM)
- ◯ **008** R. PEGNETTER ..... (2PM)
- ◯ **009** N. FLORES ..... (2PM)
- ◯ **011** V. RADHAKRISHNAN ..... (10AM)
- ◯ **012** J. SHAW ..... (12PM)
- ◯ **013** R. CLELLAND ..... (1PM)
- ◯ **014** J. FUHRMANN ..... (1PM)
- ◯ **015** N. FLORES ..... (3PM)
- ◯ **017** J. PEARSON ..... (10AM)

Show all your work for the first seven questions.
Answers out of the blue and without any supporting work will receive no credit *even if they are right!*

Write clearly. **Box your final answers.**
No calculators allowed. **No cheat sheets allowed.**

DO NOT WRITE ON THIS BOX!

problem	points	score
1	30 pts	
2	16 pts	
3	14 pts	
4	12 pts	
5	14 pts	
6	10 pts	
7	8 pts	
Mult. Choice	96 pts	
TOTAL	200 pts	

DO NOT WRITE ON THIS BOX!

1. (30 pts) Evaluate the following integrals:

(a) $\int x^2(3-x)\,dx$

(b) $\int \frac{3}{1+x^2}\,dx$

(c) $\int_0^{\pi/12} \sin(2x)\,dx$

(d) $\int \frac{x+3}{x}\,dx$

(e) $\int \frac{\sin x}{\cos^2(x)}\,dx$

(f) $\int_1^4 (3\sqrt{x} - x)\ dx$

(g) $\int (2w^{54} + 13w^{11} - 45w + 3)\ dw$

(h) $\int_0^2 te^{t^2}\ dt$

(i) $\int (6\cos(u) - 7\sec^2(u))\ du$

(j) $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\ dx$

2. (16 pts) Let R be the region betwen the curves

$$y = x^3, \qquad y = 0 \qquad \text{and} \qquad x = 2.$$

In each of the following parts, set up but **do not evaluate** the integral you would use to compute the volume of the solid in question.

(a) The solid obtained by revolving the region R about the **x–axis**, using the **disk/washer** method;

(b) The solid obtained by revolving the region R about the **x–axis**, using the **shell** method;

(c) The solid obtained by revolving the region R about the **y–axis**, using the **disk/washer** method;

(d) The solid obtained by revolving the region R about the **y–axis**, using the **shell** method.

3. (14 pts) Find the **total** area (not the net signed area) between the curve the curve $y = e^x - 1$ and the x-axis on the interval $[-1, 1]$. (Hint: You may want to sketch the graph.)

4. (12 pts) Given that $F(x) = \int_0^x \cos(t^2)\, dt$ what is $F'(\sqrt{\frac{\pi}{2}})$ and $F''(\sqrt{\frac{\pi}{2}})$?

5. (14 pts) Find the area bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

6. (10 pts) Suppose that a point moves along a curve $y = f(x)$ in the xy-plane in such a way that at each point (x, y) on the curve the tangent line has a slope of $4x^3$. Find the equation of the curve given that it passes through the point $(1, 3)$.

7. (8 pts) Complete the following theorems.

(a) **Fundamental Theorem of Calculus:** If f is _____ on $[a, b]$ and F is an _____ of f on $[a, b]$, i.e. $F'(x) = f(x)$, then

$$\int_a^b \_\_\_\_\_ dx = \_\_\_\_\_.$$

(b) **Intermediate Value Theorem:** If f is _____ on a closed interval $[a, b]$ and k is any number between _____ and _____, inclusive, then there is at least one number c in the interval $[a, b]$ such that _____.

8. Given that $\int_{-1}^{1} f(x)\,dx = 4$, $\int_{1}^{5} f(x)\,dx = -1$, and $\int_{-1}^{5} g(x)\,dx = -3$, which of the following is true?

(a) $\int_{5}^{-1} f(x)\,dx = 1$

(b) $\int_{-1}^{5} [f(x) - g(x)]\,dx = 0$

(c) $\int_{5}^{1} 2f(x)\,dx = 2$

(d) Both b and c are true

(e) Both a and c are true

9. Evaluate the following limit: $\lim_{n\to\infty} \frac{1}{n^2} \sum_{k=1}^{n} (3k-1)$

(a) 3/2

(b) 0

(c) -3

(d) 3

(e) ∞

10. Express the following Riemann sum as a definite integral from 0 to 3: $\lim_{\max \Delta_k \to 0} \sum_{k=1}^{n} \left(5x_k^{*2} - 7\right)\left(\frac{3}{n}\right)$

(a) $\int_0^3 (5x - 7)dx$

(b) $\int_1^4 (5x^2 - 7)dx$

(c) $\int_0^3 (5x - 7)^2 dx$

(d) $\int_0^3 3(5x^2 - 7)dx$

(e) $\int_0^3 (5x^2 - 7)dx$

11. Evaluate: $\lim_{h \to 0} \dfrac{\sin(x+h) - \sin(x)}{h} =$

(a) 0

(b) ∞

(c) $\cos(x)$

(d) $\sin(x)$

(e) 1

12. Find the equation of the tangent line to the graph of $f(x) = e^{-x^2}$ at $x = 1$. Express your answer in point-slope form.

(a) $(y - e^{-1}) = -2e^{-1}(x - 1)$

(b) $(y - 1) = -2e^{-1}(x - e^{-1})$

(c) $(y - 1) = e^{-1}(x - 1)$

(d) $(y - e^{-1}) = e^{-1}(x - 1)$

(e) $y = 0$

13. Find the derivative of $f(x) = (\ln(2x^4 + 3x^2 + 1))^{11}$. Don't simplify.

(a) $f'(x) = 11(\ln(8x^3 + 6x))^{10}$

(b) $f'(x) = \left(\dfrac{11}{2x^4 + 3x^2 + 1}\right)(\ln(2x^4 + 3x^2 + 1))^{10}$

(c) $f'(x) = 11(\ln(2x^4 + 3x^2 + 1))^{10} \ln\left(\dfrac{1}{2x^4 + 3x^2 + 1}\right)$

(d) $f'(x) = 11(\ln(2x^4 + 3x^2 + 1))^{10}\left(\dfrac{8x^3 + 6x}{2x^4 + 3x^2 + 1}\right)$

(e) $f'(x) = 11(\ln(2x^4 + 3x^2 + 1))^{10} \ln(2x^4 + 3x^2 + 1)$

14. A ladder 25 feet long that was leaning against a vertical wall begins to slip. Its top slides down the wall at a rate of 4 ft/sec while its bottom moves along the level ground. How fast is the bottom of the ladder moving away from the wall when the top of the ladder is 15 feet above the ground? (The bottom of the ladder is 20 feet from the wall at that point.)

(a) 3 ft/sec

(b) 4 ft/sec

(c) $\frac{25}{4}$ ft/sec

(d) $\frac{25}{3}$ ft/sec

(e) None of the above

15. Which answer choice correctly fills in the blanks?

Let $f(x) = x^2 - x$. An interval on which f satisfies the conditions of Rolle's Theorem is _____, and the value of c that satisfies the conclusion of Rolle's Theorem is _____.

(a) $(-\infty, \infty)$, $c = 0$.

(b) $[-1, 1]$, $c = 1$.

(c) $[-1, 0]$, $c = \frac{1}{2}$.

(d) $[0, 1]$, $c = \frac{1}{2}$.

(e) There is not enough information given to determine the correct answer.

16. Find the value of k, if possible, that will make the following function continuous:

$$f(x) = \begin{cases} x + 2k & \text{if } x \leq 1 \\ kx^2 + x + 1 & \text{if } x > 1 \end{cases}$$

(a) 1

(b) -1

(c) 2

(d) -2

(e) No such k exists.

17. The absolute minimum value of the function $f(x) = \frac{1}{2}x^4 - x^2 - \frac{3}{2}$ is:

(a) f does not have an absolute minimum.

(b) -1

(c) $-\frac{3}{2}$

(d) 0

(e) -2

18. The limit $\lim_{x \to +\infty} \left(\frac{\ln x}{x} \right)$

(a) Is equal to 1.

(b) ∞

(c) Is equal to 0.

(d) Is equal to $\frac{1}{2}$ because x grows slower than $\ln x$.

(e) Doesn't exist.

19. $\frac{d}{dx}\left(e^{11}\right)$ equals

(a) $11e^{10}$.

(b) 0.

(c) $\frac{e^{12}}{12}$.

(d) $\lim_{h\to 0} \frac{e^{11+h} - e^{11}}{h}$

(e) e^{11}.

20. Which of the following statements is true?

(a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

(b) If f is differentiable at $x = a$, then f is continuous at $x = a$.

(c) If $f'(a) = g'(a)$, then $f(a) = g(a)$.

(d) Both (a) and (b).

(e) All of the above.

21. Find $f''(x)$ if $f(x) = x\tan(x)$.

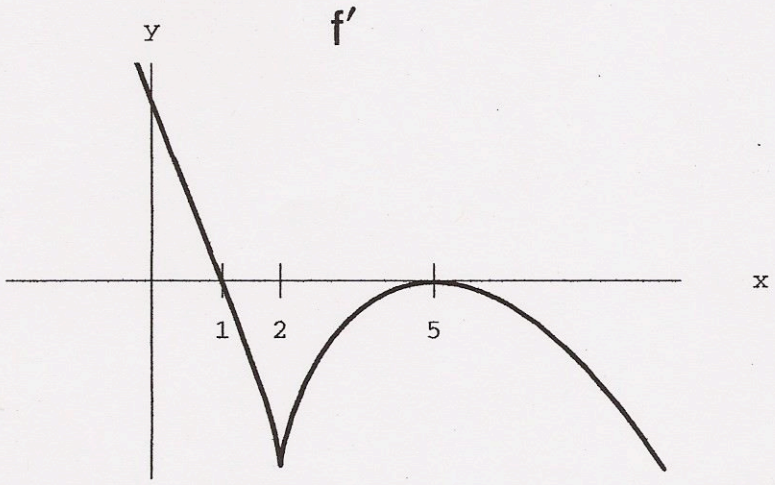
(a) $2\sec^2(x)(1 + x\tan(x))$

(b) $2x\sec^2(x)\tan(x)$

(c) $2x\sec(x) + 2\sec^2(x)$

(d) $2x\sec(x)\tan(x)$

(e) $x(\sec(x)\tan(x))^2 + \sec^2(x)$



22. The graph of the **derivative** f' of a function is shown.

Which of the following best describes what **must** be true?

(a) f is increasing on $(2, 5)$ and decreasing on both $(-\infty, 2)$ and $(5, +\infty)$.

(b) f has a relative maximum at $x = 1$.

(c) f is continuous everywhere.

(d) Both (a) and (b).

(e) Both (b) and (c).

23. Evaluate: $\lim_{x \to 1^-} \frac{1+x^2}{1-x} =$

(a) 2

(b) ∞

(c) $-\infty$

(d) Limit does not exist

(e) None of the above