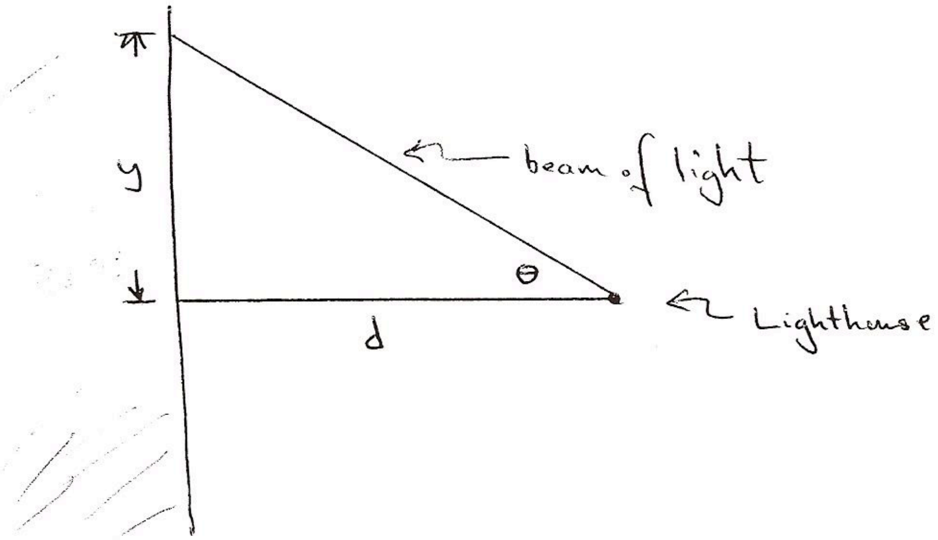
NAME: Solution

HOMEWORK FOR WORKSHEET 10 MATH 1300 DUE March 21, 2008

1. The light of an offshore lighthouse, as drawn below, is rotating at a constant rate.



Show that, as the beam of light moves down the shoreline, it moves most slowly at the point on the shore directly opposite the lighthouse. (Hint: Find an equation relating $\frac{d\theta}{dt}$ and $\frac{dy}{dt}$ and use it to minimize $\frac{dy}{dt}$.)

$\tan\theta = \frac{y}{d}$ where d is fixed

so $\sec^2(\theta)\cdot\frac{d\theta}{dt} = \frac{1}{d}\frac{dy}{dt}$

$\therefore \frac{dy}{dt} = \sec^2(\theta)\Big[\underbrace{d\,\frac{d\theta}{dt}}\Big]$

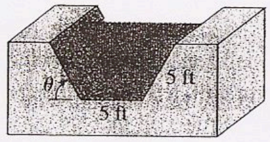
This is a positive constant in the problem so $\frac{dy}{dt}$ is smallest when $\sec^2\theta$ is smallest

$\sec^2\theta = 1 \Rightarrow \sec\theta = \pm 1 \Rightarrow \theta = 0$ or $\underbrace{\theta = \pi}_{\text{not in domain}}$

$\Rightarrow$ minimum when $\theta = 0$

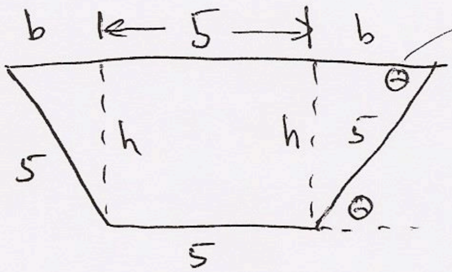
Question:
How do we know this isn't a max?

2. (Problem 39 from text, page 320) A drainage channel is to be made so that its cross section is a trapezoid with equally sloping sides. If the sides and bottom all have a length of 5 feet, how should the angle θ, with $0 \le \theta \le \pi/2$ be chosen to yield the greatest cross-sectional area of the channel?



Cross-sectional view:

Question:
Why is the angle also θ?



$$\text{Area} = 5 \cdot h + \tfrac{1}{2}bh + \tfrac{1}{2}bh$$
$$= 5h + bh$$

we want to relate h & θ and b & θ

h & θ:

$$\sin\theta = \frac{h}{5} \Rightarrow h = 5\sin\theta$$

$$\cos\theta = \frac{b}{5} \Rightarrow b = 5\cos\theta$$

$$\Rightarrow A = 25\sin\theta + 25\sin\theta\cos\theta = 25\sin\theta + \tfrac{25}{2}\sin 2\theta$$

$$\frac{dA}{dt} = 25\cos\theta + 25\cos 2\theta$$

critical point when $\cos\theta = -\cos 2\theta$, which happens

when $2\theta - \frac{\pi}{2} = \frac{\pi}{2} - \theta \Rightarrow 3\theta = \pi \Rightarrow \theta = \pi/3$