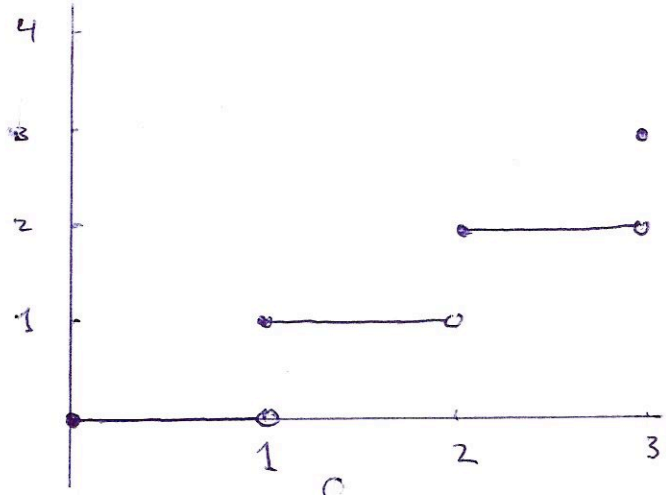
NAME: Solution

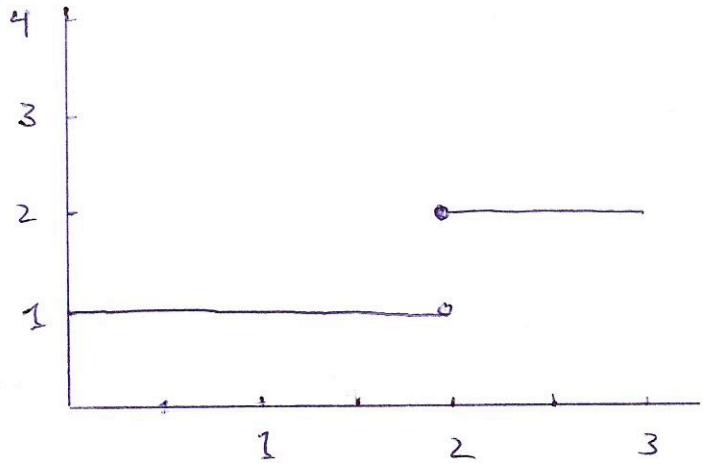
HOMEWORK FOR WORKSHEET 13 MATH 1300 DUE April 18, 2008

1. If $[[x]]$ denotes the greatest integer less than or equal to x, let $f(x) = [[x]]$ and $g(x) = [[\frac{1}{2}x]]$

(a) Graph each of the functions $f(x)$ and $g(x)$, over the interval $[0, 3]$.



$f(x)$



$g(x)$

(b) Using the graphs of part (a) evaluate each of the following integrals.

(i) $\int_0^3 f(x)dx.$ $= 0 + 1 + 2 = 3$

(ii) $\int_0^3 (f(x) + g(x))dx.$ $= \int_0^3 f(x)\,dx + \int_0^3 g(x)\,dx$

$= 3 + (2 + 2)$

$= 7$

(iii) $\int_0^3 f(x)g(x)dx.$ $= \int_0^1 f(x)g(x)\,dx + \int_1^2 f(x)g(x)\,dx + \int_2^3 f(x)g(x)\,dx$

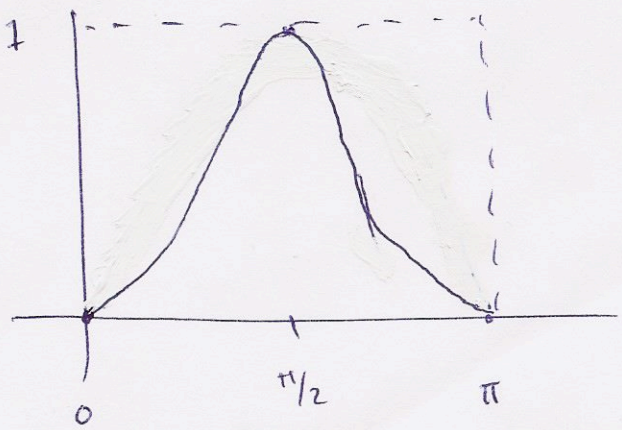
$= \int_0^1 0 \cdot 1\,dx + \int_1^2 1 \cdot 1\,dx + \int_2^3 2 \cdot 2\,dx$

$= 0 + 1 + 4$

$= 5$

2. Four calculus students disagree on the value of the integral $\int_0^{\pi} \sin^8(x)dx$. Jack says that it is equal to π. Joan says it is equal to $\frac{35\pi}{128}$. Ed claims it is equal to $\frac{3\pi}{90} - 1$ while Lesley says it is equal to $\frac{\pi}{2}$. One of them is right. Which one is it?

Hint: Do not try to evaluate the integral; try instead to eliminate the three wrong answers. (Note: If you simply use your calculator to evaluate the integral you will not be able to do the problem like this on the FInal Exam; try to think it out.)



The graph looks something like the one on the left

- Jack's answer of π is not right since the area of the indicated rectangle is π

- Ed's answer is negative, while this integral is positive.

- We are left to choose between the answers $\frac{\pi}{2}$ and $\frac{35}{128}\pi$. Since the area under the curve is less than half of the area of the rectangle, so less than $\frac{\pi}{2}$, the correct answer must be $\frac{35}{128}\pi$.