MATH 1300: Calculus I, Spring 2008
Review for Midterm Exam 1

Midterm Exam 1 covers material from sections 1.1, 1.3, 1.5, 1.6, 1.7, 2.1, 2.2, 2.3, 2.5, Appendix A, 2.6, and 3.1 of our textbook and material from Worksheets 1, 2, and 3. This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to this review. Problems are separated by section. Note that trig concepts from Appendix A appear in multiple sections. Partial solutions to the review may be posted sometime during the next few days.

CONCEPTS

1. True or False? Justify your answer.

   (a) The functions $u(x) = \dfrac{\sqrt{x}(x+1)}{x^2+x}$ and $v(x) = \dfrac{1}{\sqrt{x}}$ are equal.

   (b) $f \circ g = g \circ f$ holds for arbitrary functions f and g.

   (c) If a horizontal line does not intersect the graph of a function f, then f does not have an inverse function.

   (d) If $1 < a < b$, then $\log_a 2 > \log_b 2$.

   (e) If a function $f(x)$ does not have a limit as x approaches a from the left, then $f(x)$ does not have a limit as x approaches a from the right.

   (f) The function $f(x) = x\cos\left(\dfrac{1}{x}\right)$ is continuous everywhere.

   (g) If a function is not continuous at $x = c$, then either it is not defined at $x = c$ or it does not have a limit as x approaches c.

   (h) If $f(x)$ is a function such that $f(0) < 0$ and $f(2) > 0$, then there is a number c in the interval $(0, 2)$ such that $f(c) = 0$.

   (e) If $h(x) \le f(x) \le g(x)$ for all real numbers x and $\lim_{x\to c} h(x)$ and $\lim_{x\to c} g(x)$ exist, then $\lim_{x\to c} f(x)$ also exists.

SECTION 1.1

1. Determine if each of the following rules defines y as a function of x. If not, provide a reason why.

   (a) $x^2 + 2x + y^3 = 17$

   (b) $x^2 + 2x + y^2 + 4y - 1 = 0$

   (c) $x = \sin(y)$

2. Consider the following scenario.

   > A set of 10 missiles get launched and each missile follows a set of directions that tell the missile what target to blow up. Some of the missiles land on the same target.

   Could this scenario represent a function? If so, identify what the function would be and what the domain and range are. If not, then explain why.

3. Consider the following scenario.

   > A person types a phrase into the search engine Google, hits enter, and multiple entries are returned.

   Could this scenario represent a function? If so, identify what the function would be and what the domain and range are. If not, then explain why.

4. Consider the following function.

$$f(x) = \begin{cases} -x & \text{if } -3 \leq \text{x} \leq 0 \\ x^2 & \text{if } 0 < \text{x} < 2 \\ -1 & \text{if } 2 < \text{x} \leq 5 \end{cases}$$

   (a) Sketch the graph of f.

   (b) What is the domain of f?

   (c) What is the range of f?

5. Consider the following function.

$$g(x) = \frac{x^2 - 4}{x + 2}$$

   (a) Sketch the graph of g.

   (b) What is the domain of g?

   (c) What is the range of g?

6. A camera is mounted at a point on the ground 500 meters from the base of a space shuttle launching pad. The shuttle rises vertically when launched, and the camera's angle of elevation is continually adjusted to follow the bottom of the shuttle.

   (a) Express the height x as function of the angle of elevation θ.

(b) Find the domain of the function in part (a)

7. Determine if each of the following rules defines y as a function of x and explain why or why not. If the rule does define y as a function of x, determine the domain of the function.

   (a) $x^2 + y^2 = 3$

   (b) $x^2 + y^3 = 7$

   (c) $y(x) = \begin{cases} x^2 - 7 & \text{if } x < 0 \\ \frac{1}{x+2} & \text{if } x \geq 0 \end{cases}$

   (d) $y(x) = \begin{cases} 2x + 1 & \text{if } x \geq 1 \\ 4 - x^3 & \text{if } x \leq 1 \\ 1 & \text{if } x = 0 \end{cases}$

SECTION 1.3

1. Sketch the graph of each equation.

   (a) $y = -1 + \sqrt{2 - x}$

   (b) $y = \dfrac{2x - 1}{x}$ (Hint: divide first)

   (c) $1 - e^{2x-1}$

2. Let $f(x) = \sqrt{x}$ and $g(x) = x$. Does the domain of the product function $(f \cdot f)(x)$ equal the domain of $g(x)$? Justify your answer.

3. Let $f(x)$ and $g(x)$ be functions such that:

   Domain of $f = (-\infty, +\infty)$

   Range of $f = (-1, 1)$

   Domain of $g = (-1, +\infty)$

   Range of $g = (-\infty, 0)$

   What is the domain of g composed with f?

4. If $f(x) = 8 - x$ and $g(x) = \dfrac{1}{\sqrt{x} - 1}$, find formulas for $(g \circ f)(x)$ and $(f \circ g)(x)$. What is the domain of $g \circ f$?

5. The graph of $y = a\dfrac{1}{\cos(x+b)+2} + c$ results when the graph of $y = \dfrac{1}{\cos(x)+2}$ is reflected over the x-axis, shifted 3 units to the right, and then shifted 4 units down. Find a, b, and c.

6. Find the domains of the compositions $f \circ g$ and $g \circ f$ if $f(x) = \sin^{-1} x$ and $g(x) = \ln x$.

SECTION 1.5

1. For each of the following functions, find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$.

   (a) $f(x) = \dfrac{x+1}{x-1}$

   (b) $f(x) = 3x^3 - 15$

   (c) $f(x) = \sqrt{-x}$

   (d) $f(x) = 12x^3 - 1$

2. Complete the following identities.

   (a) $\tan(\sin^{-1} x) =$

   (b) $\cos(\tan^{-1} x) =$

SECTION 1.6

1. Simplify $\ln(x^2)$.

2. Simplify the following.

   (a) $\ln(x^2)$.

   (b) $(-27)^{2/3}$

   (c) $e^{3\ln\pi}$

   (d) $\log_4 \dfrac{1}{2} + \log_4 8 + \log_4 16$

3. For each of the following, find ALL values of x which satisfy the given equation.

   (a) $\ln\left(x^2\right) = \ln x$

(b) $\ln\left(x^2\right) = (\ln x)^2$

(c) $\ln\sqrt{x} = \sqrt{\ln x}$

(d) $5^x = 4$

(e) $2^{4x+1} = 3$

(f) $\cos^2(x) - \cos(x) + 2 = 0$

4. Given the equation $2^{t/\tau} = e^{\alpha t}$, solve for the *growth constant* α in terms of τ using natural logarithms. (The parameter τ is called the *doubling time*.)

5. Explain how the graph of the logarithmic function with base $b > 0$ can be obtained from the graph of the natural logarithmic function by using one or more translations, reflections, stretches or compressions.

SECTION 2.1

1. Sketch the graph of a possible function f that has all properties (a)-(g) listed below.

   (a) The domain of f is $[-1, 2]$

   (b) $f(0) = f(2) = 0$

   (c) $f(-1) = 1$

   (d) $\lim_{x\to 0^-} f(x) = 0$

   (e) $\lim_{x\to 0^+} f(x) = 2$

   (f) $\lim_{x\to 2^-} f(x) = 1$

   (g) $\lim_{x\to -1^+} f(x) = -1$

2. Sketch the graphs of possible functions f, g, and h such that: f satisfies property (a) below, g satisfies property (b) below, and h satisfies property (c) below. There should be three separate graphs.

   (a) $\lim_{x\to 0} f(x) = 1$

   (b) $\lim_{x\to 0^-} g(x) = -1$ and $\lim_{x\to 0^+} g(x) = +1$

   (c) $\lim_{x\to 0} h(x) \neq h(0)$

SECTION 2.2

1. Find the following limits.

   (a) $\lim_{x \to 2} x^2 + 4x - 12$

   (b) $\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 + 4x + 3}$

   (c) $\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - x - 2}$

   (d) $\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 4x + 4}$

   (e) $\lim_{x \to -3} \frac{x}{x + 3}$

   (f) $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$

   (g) $\lim_{x \to -2} \frac{x^2 - 5x - 14}{x + 2}$

   (h) $\lim_{x \to 0} \frac{4x - 3}{4x^2 + 3}$

   (i) $\lim_{x \to 3^-} \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$

2. For f defined as follows, find the given limits.

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ 2x + 1 & \text{if } 0 \leq x \leq 4 \\ x^2 & \text{if } x > 4 \end{cases}$$

   (a) $\lim_{x \to 0^+} f(x)$

   (b) $\lim_{x \to 4} f(x)$

3. Let $\lim_{x \to a} f(x) = -3$, $\lim_{x \to a} g(x) = 6$, and $\lim_{x \to a} h(x) = 0$. Find the following limits, if they exist:

   (a) $\lim_{x \to a} (f(x) + 2g(x))$

   (b) $\lim_{x \to a} \frac{(g(x))^2}{f(x) + 5}$

(c) $\lim_{x \to a} \frac{2f(x)}{h(x)}$

(d) $\lim_{x \to a} \frac{7f(x)}{2f(x) + g(x)}$

(e) $\lim_{x \to a} \sqrt[3]{g(x) + 2}$

SECTION 2.3

1. Find the following limits.

   (a) $\lim_{x \to \infty} 5x^2 - 2x + 1$

   (b) $\lim_{x \to -\infty} \frac{\sqrt{3x^2 - 5}}{x - 7}$

   (c) $\lim_{x \to \infty} \frac{2}{\pi} \tan^{-1}(x)$

   (d) $\lim_{x \to \infty} \frac{\sqrt{5x^2 - 2}}{x + 3}$

   (e) $\lim_{x \to \infty} \sqrt{x^2 + 3} - x$

   (f) $\lim_{x \to 1^-} \ln(1 - x)$

SECTION 2.5

1. Find the values of x, if any, for which the following function is not continuous.

$$f(x) = \begin{cases} \frac{x}{x^2 - 1}, & x \geq 0 \\ \frac{x+3}{|x-3|}, & x < 0. \end{cases}$$

2. Find the values for k such that the following function $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} \sqrt{x^2 - 16}, & x \geq 5 \\ \frac{3k}{x-1}, & x < 5. \end{cases}$$

3. Is the following function continuous on the interval $[0, 1]$? Justify your answer.

$$f(x) = \frac{1}{x^5 + \pi x - e}$$

SECTION 2.6

1. If $3x \leq f(x) \leq x^3 + 2$ for $0 \leq x \leq 2$, evaluate $\lim_{x \to 1} f(x)$.

2. Prove that $\lim_{x \to 0} \left(x^4 \cos(2/x) \right) = 0$.

SECTION 3.1

1. Let $f(x) = x^2 - x$.

   (a) Find the slope of the tangent line at $x = 2$ using the limit definition of $m_{\tan}$.

   (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

2. Let $g(x) = |x|$. Using the limit definition of $m_{\tan}$, prove that there is not a tangent line to the graph of g at $x = 0$ (show that the slope of the tangent line does not exist at $x = 0$).