MATH 1300: Calculus I, Spring 2008
Review for Midterm Exam 2

Midterm Exam 2 covers material from sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 4.1, 4.2, 4.3, and 4.4 of our textbook and material from Worksheets 4, 5, 6, and 7. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to this review.

To study for the exam, you should do the following (in this order):

1. Study class notes and read the textbook
2. Review homework problems, worksheets, and homework associated with the worksheets
3. Attempt as many problems on this review that you feel are necessary to increase your understanding of the material
4. If you feel you need additional practice, attempt problems in the chapter reviews (that are similar to assigned homework problems) located at the end of chapters 3 and 4

I. Concepts

True or False? Justify your answers.

(a) If a function is not continuous at $x = c$, then it is not differentiable at $x = c$.

(b) The derivative with respect to x of a rational function $f(x)$ is a rational function.

(c) $\dfrac{d}{dx}[f(cx)] = c\dfrac{d}{dx}[f(x)]$.

(d) $\dfrac{d}{dx}\left[\dfrac{1}{f(x)}\right] = \dfrac{1}{f'(x)}$.

(e) If f and g are differentiable on an open interval containing c, except possibly at $x = c$, and $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}$ is an indeterminate form of type ∞/∞, then $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$.

(f) $\dfrac{d}{dx}[(2x^3 + 5x^2 - 7)(3\cos(x) + 13x)] = (6x^2 + 10x)(-3\sin(x) + 13)$

II. Additional Problems

1. A representative from a certain publishing company is dropped from the top of a 220 foot tall building. The height $h(t)$ in feet of the representative at t seconds is given by the position function $h(t) = -16t^2 - 26t + 220$.

(a) Find the average velocity of the representative between 2 seconds and 4 seconds. Label your answer with the appropriate units.

(b) Find the instantaneous velocity of the representative at 2 seconds. Label your answer with the appropriate units.

(c) What does the sign of your answer in part (b) mean?

2. Let $f(x) = x^2 - x$. Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

3. Let $y = x^2 \sin\left(\frac{\pi}{4}x\right)$. Find $\frac{dy}{dx}$.

4. Find $\frac{d}{dx}[\sec(x)\tan(x) + (\cos(x))^2]$

5. Find the equation for the line tangent to the graph of $f(x) = \csc(2x) + \tan(x)$ at the point $(\frac{\pi}{4}, 2)$.

6. If $f(1) = 3$, $g(5) = 2$, $h(1) = 5$, $f'(2) = 3$, $g'(5) = -2$, and $h'(1) = -8$, find $(f \circ g \circ h)'(1)$

7. A spherical balloon is deflated so that its volume decreases at a constant rate of 3 in^3/sec. How fast is the balloon's diameter decreasing when the radius is 2 inches?

8. A 10-foot ladder is leaning against a building. If the top of the ladder slides down the wall at a constant rate of 2 feet per second, how fast is the acute angle the ladder makes with the ground decreasing when the top of the ladder is 5 feet from the ground? (Give the answer in radians per second.)

9. Let $y = \arcsin(x)$. Find the equation of the tangent line at $x = \frac{1}{2}$.

10. Let $f(x) = \frac{x}{x^2 + 3}$.

(a) Use the quotient rule to calculate $f'(x)$

(b) Calculate $f'(x)$ using the product and the chain rule (Hint: Rewrite $f(x)$ as $f(x) = x(x^2 + 3)^{-1}$.)

(c) Find $f'(x)$ using logarithmic differentiation.

11. Find $\frac{dy}{dx}$ for each of the following:

(a) $y = \ln x^2$

(b) $y = x \ln 2$

(c) $y = x^e$

(d) $y = x^x$

(e) $y = 2^x$

(f) $y = \ln(\cos x)$

(g) $y = e^{3x+5}$

12. Use logarithmic differentiation to find $f'(x)$ for each of the following.

(a) $f(x) = \frac{(3-x)^{1/3}x^2}{(1-x)(3+x)^{2/3}}$.

(b) $f(x) = (x+1)(e^{x^2}+1)$

13. Evaluate each of the following limits:

(a) $\lim_{x\to\pi/2} \dfrac{1-\sin(x)}{\cos(x)}$

(b) $\lim_{x\to 0} \dfrac{e^x - 1}{x^3}$

(c) $\lim_{x\to 0^+} x \ln x$

(d) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$

(e) $\lim_{x\to 0^+} x^x$

14. Let $f(x) = |x|$ and $g(x) = \sin x$. Can L'Hôpital's Rule be used to evaluate $\lim_{x\to 0} \frac{f(x)}{g(x)}$? If so, what is the limit? If not, justify your answer.

III. Even More Problems

If you feel that you still need additional practice, here are more problems to try.

Section 3.1: 10, 20

Section 3.2: 7, 22, 23

Section 3.3: 5, 11, 63

Section 3.4: 13, 27

Section 3.5: 7, 15, 25

Section 3.6: 13, 21, 33

Section 3.7: 15, 37

Section 4.1: 12, 23

Section 4.2: 5, 13, 21, 29, 39

Section 4.3: 19, 33, 37, 44

Section 4.4: 7, 21, 35