Math 1300: Calculus I, Spring 2008
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Solution to Exercise 43 in Section 5.4

Let $f(x) = \sin x$ and $g(x) = x$. Define a third function

$$h(x) = g(x) - f(x) = x - \sin x.$$

We will minimize h on the interval $[0, 2\pi]$. We see that

$$h'(x) = 1 - \cos x.$$

We can find the critical points by solving for x in $h'(x) = 0$. We see that

$$\begin{aligned} 0 &= 1 - \cos x \\ \cos x &= 1 \\ x &= 2k\pi, \end{aligned}$$

where k can be any integer. The only critical points from above that are also in the interval $[0, 2\pi]$ are 0 and 2π. Since h is continuous on $[0, 2\pi]$, the absolute maximum and the absolute minimum must occur at there two x-values. We see that $h(0) = 0 - \sin 0 = 0$ and $h(2\pi) = 2\pi - \sin 2\pi = 2\pi$. So, the absolute minimum of 0 occurs at $x = 0$. Since the absolute minimum is 0, h must take values that are greater than or equal to 0 on the rest of the interval. This shows that $h(x) \geq 0$ on $[0, 2\pi]$. In other words, $x - \sin x \geq 0$ on $[0, 2\pi]$. This implies that $x \geq \sin x$ on $[0, 2\pi]$, which is what we wanted to prove.