WORKSHEET 1 **MATH 1300** **JANUARY 17, 2008**

Goal: To understand the relationship between the functions $f(x)$ and $f^{-1}(x)$ both graphically and, for some functions, algorithmically.

1. Sometimes a function can be described by a simple algorithm.

(a) Consider the following algorithm:

Step 0 Take any real number x
Step 1 Add 1 to x
Step 2 Square the result of Step 1
Step 3 Add 2 to the result of Step 2

Let y denote the result of the above algorithm. Express y as a function of x.

(b) Consider the two algorithms:

Algorithm 1:

Step 0 Take a real number x
Step 1 Square x
Step 2 Add 1 to the result of Step 1

Algorithm 2:

Step 0 Take any real number x
Step 1 Subtract 1 from x
Step 2 Take the positive square root of the result of Step 1

(i) Check whether these two algorithms are inverses of one another. That is, does Algorithm 1 followed by Algorithm 2 (and Algorithm 2 followed by Algorithm 1) *undo* each other? (Make a flow chart.)

Algorithm 1 followed by Algorithm 2:

Algorithm 2 followed by Algorithm 1:

(ii) Express Algorithm 1 as a function of the form $y = f(x)$ and Algorithm 2 as a function of the form $y = g(x)$, and graph them.

(iii) Are f and g inverses of each other? If yes, explain why. If no, can either of their domains be restricted so that they are inverses of each other?

2. For a function $g(x)$ we let $\Gamma(g)$ denote the graph of $g(x)$. That is,

$$
\begin{aligned}
\Gamma(g) &= \text{the collection of all points on the graph of } g(x) \\
&= \text{the collection of all pairs of numbers } (x, y) \text{ where } y = g(x) \\
&= \{(x, y) : y = g(x)\}
\end{aligned}
$$

(a) Suppose f is a function with an inverse function f^{-1}. Let $P = (a, b)$ be a point of $\Gamma(f)$. Explain why $P' = (b, a)$ is a point on $\Gamma(f^{-1})$.

(b) What does the result of part (a) tell us about the relationship between $\Gamma(f)$ and $\Gamma(f^{-1})$? Hint: What is the relationship between (a, b) and (b, a) in the coordinate plane?

3. (This is an extension of Problem 2) Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are two points on $\Gamma(f)$, and let L denote the line through P and Q. (L is called a *secant line* or a *chord* to $\Gamma(f)$, or simply to f.) We saw in Problem 2 that $P' = (y_1, x_1)$ and $Q' = (y_2, x_2)$ lie on $\Gamma(f^{-1})$. Let L' be the secant line to $\Gamma(f^{-1})$ through P' and Q'. What, if any, is the relationship between the slope of L and the slope of L'?