WORKSHEET 12 **MATH 1300** **April 10, 2008**

Goal: To study the area under a curve without directly using the area function $A(x)$.

1. The United States Postal Service used to charge postage for a first class domestic letter as follows: The postage was 29 cents for the first ounce and 23 cents for each additional ounce, up to 11 ounces. Therefore, if $C(w)$ is the cost of mailing a letter weighing w ounces, then:

$$C(w) = \begin{cases} 29 & \text{if } 0 \leq w \leq 1 \\ 52 & \text{if } 1 < w \leq 2 \\ 75 & \text{if } 2 < w \leq 3 \\ \text{etc.} & \end{cases} \tag{1}$$

(a) Draw a graph of C as a function of w (on the x-axis use a scale of 0 to 6 and on the y-axis use a scale of 0 to 150).

(b) What is the total cost of mailing 5 letters if their weights are .5, 1.5, 2.5, 3.5, and 4.5 ounces, respectively?

(c) Can you describe geometrically (more specifically, using areas) your answer to part (b)? (Refer to the graph in part (a)).

2. (a) Provide a fairly accurate graph of the function $f(x) = 4 - x^2$ over the interval $[-2, 2]$.

We are going to estimate the area under the above graph, and above the $x-$axis, over the interval $[0, 2]$ by using rectangles. (The exact area is $\frac{16}{3}$ square units.)

(b) On your graph, above, divide the interval $[0, 2]$ into 4 equal subintervals and then draw 4 rectangles. Each of these 4 rectangles should have one of your four subintervals as a base and each rectangle's height should be determined by the point in the subinterval whose $y-$coordinate is minimal (so each rectangle will lie under the graph but just touch it.)

(c) Estimate the area under the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ by calculating the sum of the areas of the 4 rectangles you have drawn.

(d) If you were to repeat the above process with more than 4 rectangles do you think you would get a better or worse approximation of the area?

(e) Now test your answer to part (d). On axes below, redraw the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ and subdivide the interval into 8 equal subintervals. Using these 8 subintervals for their bases draw 8 rectangles as in part (c).

(f) Approximate the area under the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ by finding the sum of the areas of the 8 rectangles you have drawn. (And then compare this estimate with your earlier one.)

3. In Problem 2 you obtained lower estimates for the area under the curve. Describe how you could modify the process in Problem 2 to obtain upper estimates for the area under the curve.

4. Explain how the above process could, *in theory*, be used to find the exact area under the curve.