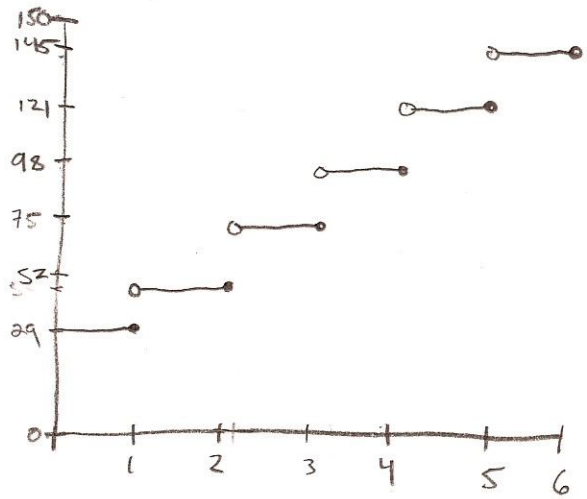
WORKSHEET 12 **MATH 1300** **April 10, 2008**

Goal: To study the area under a curve without directly using the area function $A(x)$.

1. The United States Postal Service used to charge postage for a first class domestic letter as follows: The postage was 29 cents for the first ounce and 23 cents for each additional ounce, up to 11 ounces. Therefore, if $C(w)$ is the cost of mailing a letter weighing w ounces, then:

$$C(w) = \begin{cases} 29 & \text{if } 0 \le w \le 1 \\ 52 & \text{if } 1 < w \le 2 \\ 75 & \text{if } 2 < w \le 3 \\ \text{etc.} & \end{cases} \tag{1}$$

(a) Draw a graph of C as a function of w (on the x-axis use a scale of 0 to 6 and on the y-axis use a scale of 0 to 150).



(b) What is the total cost of mailing 5 letters, one each of the following weighs, in ounces, .5, 1.5, 2.5, 3.5, and 4.5?

$C(0.5) + C(1.5) + C(2.5) + C(3.5) + C(4.5)$

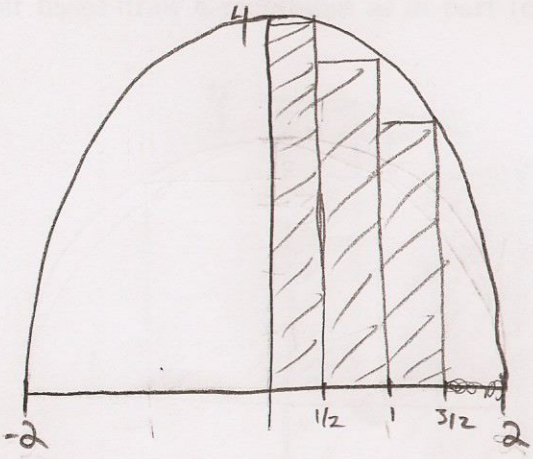
$= 29 + 52 + 75 + 98 + 121$

$= 375\ ¢ = \$3.75$

(c) Can you describe geometrically (more specifically, using areas) your answer to part (b)? (Refer to the graph in part (a).

Total Cost = Area under graph of $C(w)$ over $[0,5]$

2. (a) Provide a fairly accurate graph of the function $f(x) = 4 - x^2$ over the interval $[-2, 2]$.



We are going to estimate the area under the above graph, and above the $x-$axis, over the interval $[0, 2]$ by using rectangles. (The exact area is $\frac{16}{3}$ square units.)

(b) On your graph, above, divide the interval $[0, 2]$ into 4 equal subintervals and then draw 4 rectangles. Each of these 4 rectangles should have one of your four subintervals as a base and each rectangle's height should be determined by the point in the subinterval whose $y-$coordinate is minimal (so the rectangle will lie under the graph but just touch it.)

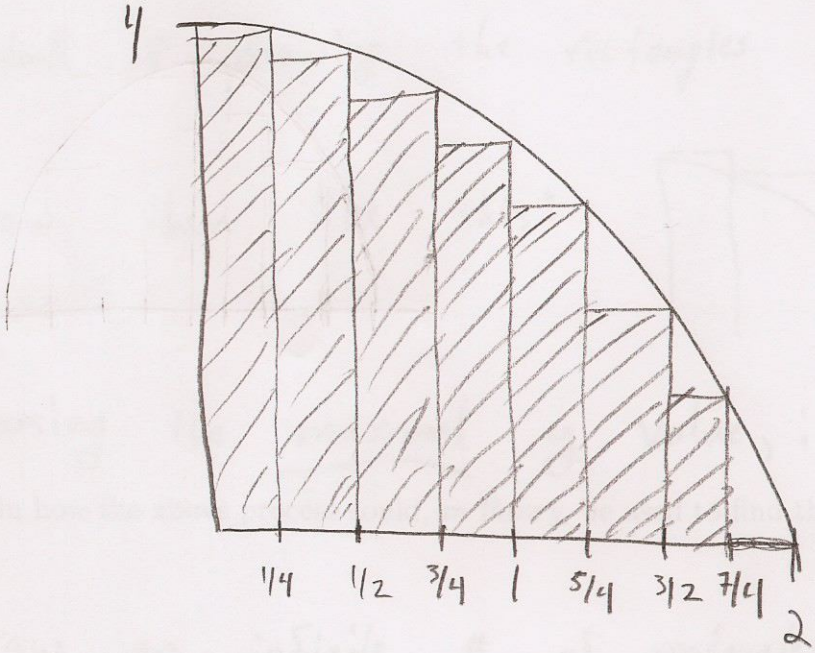
(c) Estimate the area under the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ by calculating the sum of the areas of the 4 rectangles you have drawn.

$1/2 \cdot 15/4 + 1/2 \cdot 3 + 1/2 \cdot 7/4 + 1/2 \cdot 0 = 17/4$

(d) If you were to repeat the above process with more than 4 rectangles do you think you would get a better or worse approximation of the area?

Better. A lot of area is missed by the rightmost rectangle

(e) Now test your answer to part (d). On axes below, redraw the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ and subdivide the interval into 8 equal subintervals. Using these 8 subintervals for their bases draw 8 rectangles as in part (c).



(f) Approximate the area under the graph of $f(x) = 4 - x^2$ over the interval $[0, 2]$ by finding the sum of the areas of the 8 rectangles you have drawn. (And then compare this estimate with your earlier one.)

$$1/4 \cdot 63/16 + 1/4 \cdot 15/4 + 1/4 \cdot 55/16 + 1/4 \cdot 3 + 1/4 \cdot 39/16 + 1/4 \cdot 7/4$$

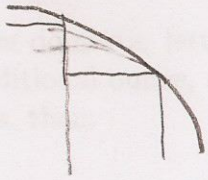
$$+ 1/4 \cdot 15/16 + 1/4 \cdot 0 = 77/16$$

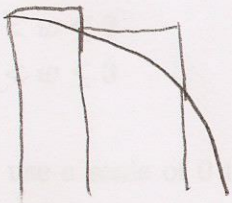
Earlier: $17/4 = 4.25$

Now: $77/16 = 4.8125$ ← Better!

Actual: $16/3 = 5.3\overline{3}$

3. In Problem 2 you obtained lower estimates for the area under the curve. Describe how you could modify the process in Problem 2 to obtain upper estimates for the area under the curve.

Instead of drawing the rectangles like 

draw them like this:  do this by

choosing the <u>maximal</u> y value, instead of the minimal one.

4. Explain how the above process could, *in theory*, be used to find the exact area under the curve.

Draw an infinite # of rectangles <u>under</u> the curve and add up area. Call this number U.

Draw an infinite # of rectangles <u>above</u> the curve and add up the area. Call this number A.

If U = A then we have found the exact area under the curve! However, it is easy to imagine situations where U ≠ A (although these may not be considered "curves").