WORKSHEET 1 **MATH 1300** **JANUARY 17, 2008**

Goal: To understand the relationship between the functions $f(x)$ and $f^{-1}(x)$ both graphically and, for some functions, algorithmically.

1. Sometimes a function can be described by a simple algorithm.

(a) Consider the following algorithm:

Step 0 Take any real number x

Step 1 Add 1 to x

Step 2 Square the result of Step 1

Step 3 Add 2 to the result of Step 2

Let y denote the result of the above algorithm. Express y as a function of x.

Solution: $y = (x+1)^2 + 2$

(b) Consider the two algorithms:

Algorithm 1:

Step 0 Take a real number x

Step 1 Square x

Step 2 Add 1 to the result of Step 1

Algorithm 2:

Step 0 Take any real number x

Step 1 Subtract 1 from x

Step 2 Take the positive square root of the result of Step 1

(i) Check whether these two algorithms are inverses of one another. That is, does Algorithm 1 followed by Algorithm 2 (and Algorithm 2 followed by Algorithm 1) *undo* each other? (Make a flow chart.)

Algorithm 1 followed by Algorithm 2:

$$\underbrace{x \longrightarrow x^2 \longrightarrow x^2+1}_{\text{Algorithm 1}} \underbrace{\longrightarrow x^2 \longrightarrow |x|}_{\text{Algorithm 2}}$$

They undo each other if $x \geq 0$

Algorithm 2 followed by Algorithm 1:

$$\underbrace{x \longrightarrow x-1 \longrightarrow \sqrt{x-1}}_{\text{Algorithm 2 (requires } x \geq 1)} \underbrace{\longrightarrow x-1 \longrightarrow x}_{\text{Algorithm 1}}$$

They undo each other if $x \geq 1$

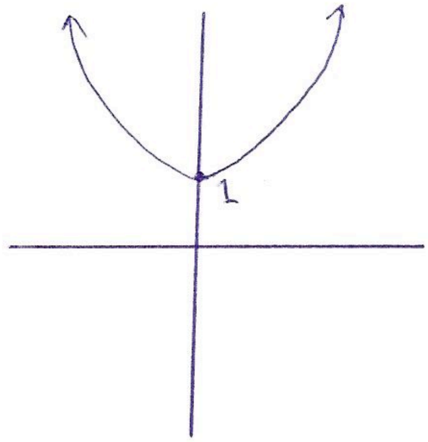
(ii) Express Algorithm 1 as a function of the form $y = f(x)$ and Algorithm 2 as a function of the form $y = g(x)$, and graph them.

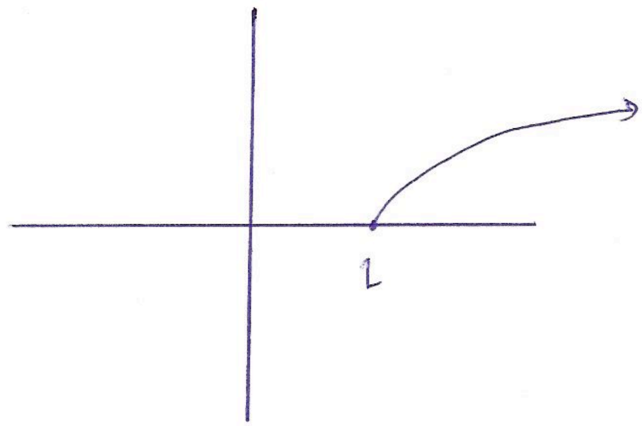
Algorithm 1

$f(x) = x^2 + 1$

Algorithm 2

$g(x) = \sqrt{x-1}$





(iii) Are f and g inverses of each other? If yes, explain why. If no, can either of their domains be restricted so that they are inverses of each other?

No, but if we restrict the domain of $f(x)$ to the set $\{x : x \geq 0\}$ they are. (See page 53 of text)

2. For a function $g(x)$ we let $\Gamma(g)$ denote the graph of $g(x)$. That is,

$$\begin{aligned}
\Gamma(g) &= \text{the collection of all points on the graph of } g(x) \\
&= \text{the collection of all pairs of numbers } (x, y) \text{ where } y = g(x) \\
&= \{(x, y) : y = g(x)\}
\end{aligned}$$

(a) Suppose f is a function with an inverse function f^{-1}. Let $P = (a, b)$ be a point of $\Gamma(f)$. Explain why $P' = (b, a)$ is a point on $\Gamma(f^{-1})$.

$(a, b) \in \Gamma(f) \Rightarrow f(a) = b \Rightarrow f^{-1}(b) = a \Rightarrow (b, a) \in \Gamma(f^{-1})$

(b) What does the result of part (a) tell us about the relationship between $\Gamma(f)$ and $\Gamma(f^{-1})$? Hint: What is the relationship between (a, b) and (b, a) in the coordinate plane?

Since (a, b) and (b, a) are symmetric with respect to the line $y = x$, $\Gamma(f)$ and $\Gamma(f^{-1})$ are symmetric with respect to that line.

3. (This is an extension of Problem 2) Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are two points on $\Gamma(f)$, and let L denote the line through P and Q. (L is called a *secant line* or a *chord* to $\Gamma(f)$, or simply to f.) We saw in Problem 2 that $P' = (y_1, x_1)$ and $Q' = (y_2, x_2)$ lie on $\Gamma(f^{-1})$. Let L' be the secant line to $\Gamma(f^{-1})$ through P' and Q'. What, if any, is the relationship between the slope of L and the slope of L'?

The slope of L is $\dfrac{y_2 - y_1}{x_2 - x_1}$

The slope of L' is $\dfrac{x_2 - x_1}{y_2 - y_1}$

Those numbers are reciprocals of each other.