WORKSHEET 2 **MATH 1300** **JANUARY 24, 2008**

Goal: To understand rational functions where both the numerator and denominator vanish.

The key to analyzing a rational function $f(x) = \frac{P(x)}{Q(x)}$ at, or near, $x = a$ when $P(a) = 0$ and $Q(a) = 0$ is the following theorem from algebra:

The Factor Theorem. Suppose $P(x)$ is a polynomial and a is a root of $P(x)$. That is, $P(a) = 0$. Then $P(x)$ factors as $P(x) = (x - a)Q(x)$, where $Q(x)$ is another polynomial.

For example, let $P(x) = x^3 + 8$. It's easy to see that $P(-2) = 0$, since

$$P(-2) = (-2)^3 + 8 = -8 + 8 = 0,$$

so by the above theorem, $x - (-2) = x + 2$ is a factor of $P(x)$. Using long division we can EXPLICITLY factor $x + 2$ out of $P(x)$ to find that $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$.

1. Use the Factor Theorem to simplify the rational function $f(x) = \dfrac{x^2 - x - 2}{x + 1}$ and discover that its graph is a line missing a single point.

2. Use the Factor Theorem to see how to algebraically simplify each of the following rational functions. Sketch the graph for each function.

a. $f(x) = \dfrac{x^3 + x^2}{x + 1}$

b. $g(x) = \dfrac{x^3 - 6x^2 + 12x - 8}{x - 2}$

3. Use the graphs on the previous page to evaluate each of the following limits:

a. $\lim_{x \to -1} \dfrac{x^3 + x^2}{x + 1}$

b. $\lim_{x \to 2} \dfrac{x^3 - 6x^2 + 12x - 8}{x - 2}$

4. Explain how the Factor Theorem might be used to evaluate a limit $\lim_{x \to a} \frac{P(x)}{Q(x)}$, where $P(a) = 0$ and $Q(a) = 0$, without first graphing the function $f(x) = \frac{P(x)}{Q(x)}$

5. Use your observation in Problem 4 to evaluate the limit $\lim_{x \to 1} \frac{x^3 + x^2 + x - 3}{x^2 + x - 2}$.