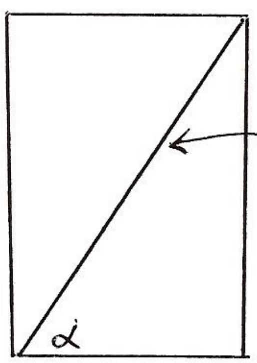
Goal: To review some information concerning trigonometric functions and to begin using the basic trigonometric functions in so-called "applied problems".

1. The red stripe on a barber pole makes one complete revolution around the pole. If the pole is 50 inches long and with the amazingly, precise radius of $\frac{25}{\pi\sqrt{3}}$ inches, what angle does the stripe make with the pole, and how long is the stripe? (Assume the stripe is a thin line.)

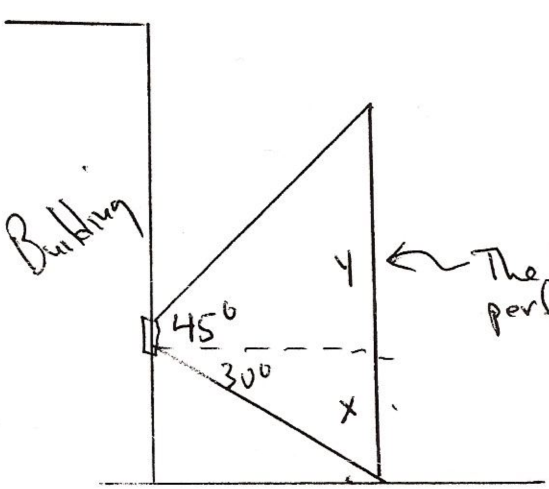
Imagine that the barber pole is a hollow cylinder that we slice from top to bottom and roll out flat. This will produce a rectangle (not drawn to scale):



We see from this drawing that

$$\tan\alpha = \frac{50}{\frac{50}{\sqrt{3}}} = \sqrt{3}. \quad \text{So } \alpha = 60°.$$

2. While leaning out of your apartment's window, you notice a) that is is a warm and sunny day, b) that the line of sight to the top of a nearby tree makes an angle of 45 degrees above the horizontal, and c) that the line of sight to the base of the tree makes an an angle of 30 degrees below the horizontal. Taking advantage of a), you go outside and discover that it is 40 feet from the building to the base of the tree. How tall is the tree?



$$\tan 30° = \frac{x}{40} \quad \& \quad \tan 30° = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \frac{x}{40} = \frac{\sqrt{3}}{3} \Rightarrow x = 40\frac{\sqrt{3}}{3}$$

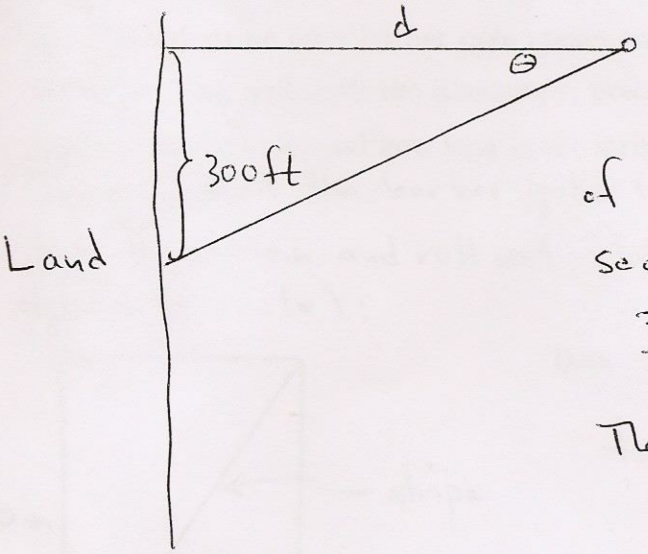
$$\tan 45° = \frac{y}{40} \quad \& \quad \tan 45° = 1$$

$$\Rightarrow \frac{y}{40} = 1 \Rightarrow y = 40$$

So the height of the tree is $x+y$

$= \frac{40\sqrt{3}}{3} + 40$ ft

3 a. A lighthouse sits on a rock offshore and its beam rotates once every 48 seconds. Starting from the point on the shore nearest the lighthouse, the beam moves 300 feet down the shore in 4 seconds. How far is the lighthouse from the shore?



The angle θ corresponds to the number of degrees the light will turn in 4 seconds. So we have the proportion:

$$\frac{360^\circ}{48 \text{ secs}} = \frac{\theta^\circ}{4 \text{ secs}} \Rightarrow \theta = \frac{360}{12} = 30^\circ$$

Then

$$\tan 30^\circ = \frac{300}{d} \Rightarrow d = 300\sqrt{3}$$

3 b. Discuss *qualitatively* (i.e., without actually solving the problem) whether the lighthouse would be nearer to or farther from the shore, relative to your answer in 3 a, if the *only* change to the above information is:

1. the beam moves *500* feet down the shore in 4 seconds

   In the drawing above the triangle has the same shape but the 300 would be replaced by 500. Then d is larger.

2. the beam rotates once every *90* seconds

   In this case the angle in the above triangle will be less than 30° since the light is rotating more slowly. Then d is larger.

3. the beam moves 300 feet down the shore in 6 seconds

   In this case the angle is larger so d must be smaller