

Solution

WORKSHEET 4

MATH 1300

February 7, 2008

Goal: To explore the meaning of the *average rate of change* of a function.

1. Amy takes a trip from Boulder to Summit County. Due to road construction, she drives the first 10 miles at a constant speed of 20 mph. For the next 40 miles she maintains a constant speed of 60 mph and then stops at Starbucks for 10 minutes to buy her third skinny, grande latte of the morning. She then drives the next 45 miles at a constant speed of 45 mph.

a. Draw a graph which shows her distance, in miles, along the road from her starting point as a function of time, in minutes. (Before drawing the graph determine how long Amy's trip took and how far she travelled.)

Amy's trip can be viewed in 4 pieces:

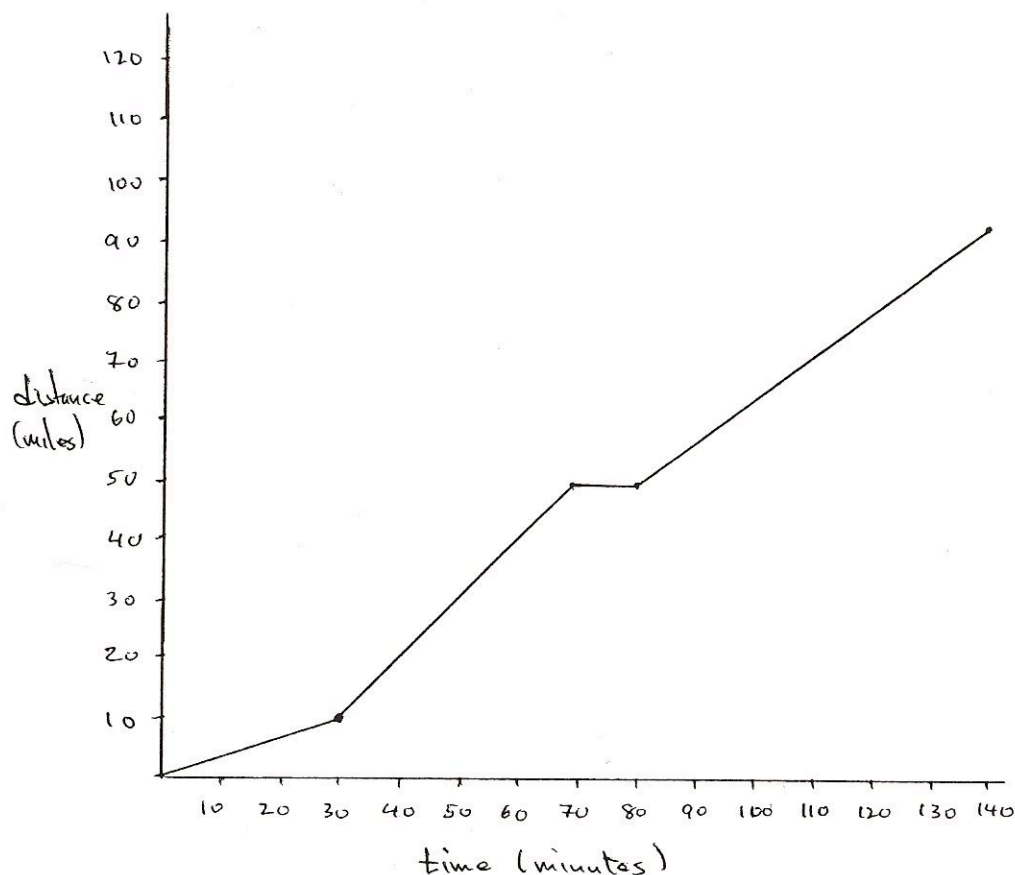
The first 10 miles (at 20 mph this takes 30 minutes)

The next 40 miles (at 60 mph this takes 40 minutes)

The stop at Starbucks 10 minutes

The last 45 miles (at 45 mph this takes 60 minutes)

Total distance 95 miles Total time 140 minutes



b. Find the slope of each line segment in the above graph and compare it with Amy's velocity during the corresponding time intervals.

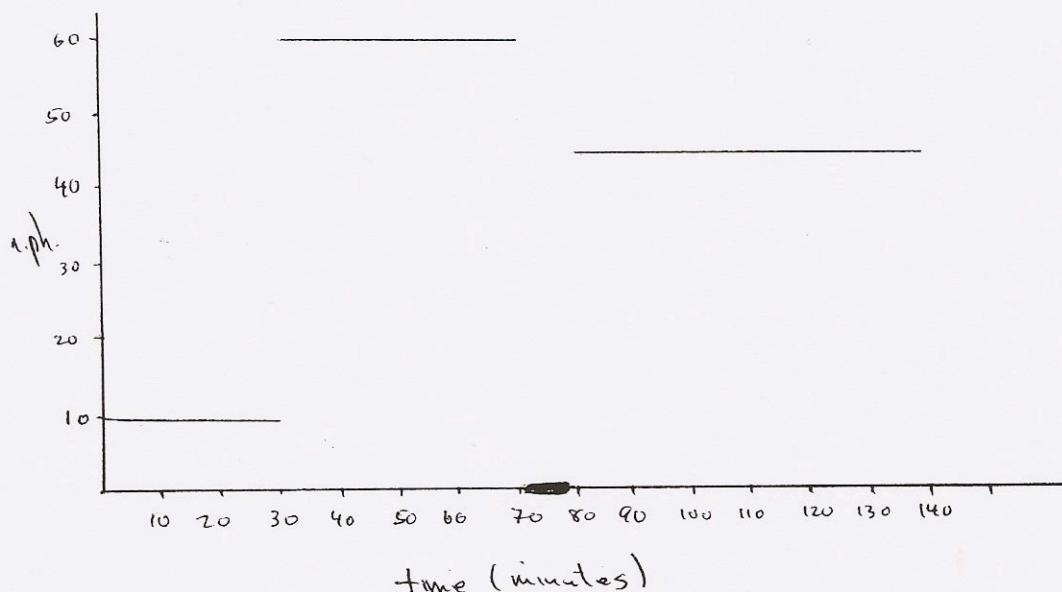
First segment: slope = $\frac{10}{30}$ (miles per minute) \leadsto 20 mph

Second segment: slope = $\frac{50-10}{70-30} = \frac{40}{40} = 1$ (mile per min) \leadsto 60 mph

Third segment: slope = 0 \leadsto 0 mph

Fourth segment: slope = $\frac{95-50}{140-80} = \frac{45}{60}$ (miles per min) \leadsto 45 mph

c. Draw a graph which shows her velocity as a function of time.



d. Explain why your graph for Amy's velocity as a function of time makes our description of Amy's trip unrealistic.

The velocity graph should be continuous.

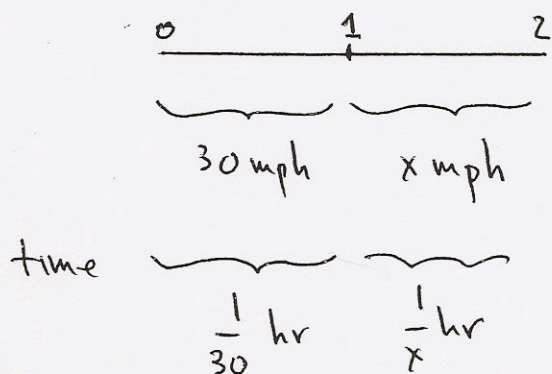
2 a. A car travels for 30 miles with an average velocity of 40 mph, and then for 30 minutes at 60 mph. What is the average velocity of the car for the entire trip?

$$\begin{aligned} \text{The average velocity} &= \frac{\text{total distance}}{\text{total time}} = \frac{30 \text{ miles} + 30 \text{ miles}}{\frac{3}{4} \text{ hr} + \frac{1}{2} \text{ hr}} \\ &= \frac{60}{\frac{5}{4}} \text{ mph} \\ &= 48 \text{ mph} \end{aligned}$$

b. Another car travels for 30 minutes at 40 mph and then for 30 minutes at 60 mph. Find the average velocity of this car for the one-hour period.

$$\begin{aligned} \text{The average velocity} &= \frac{20 \text{ miles} + 30 \text{ miles}}{\frac{1}{2} \text{ hr} + \frac{1}{2} \text{ hr}} \\ &= 50 \text{ mph} \end{aligned}$$

c. A car is to travel two miles. It travels the first mile at an average velocity of 30 mph. The mathematically obsessed driver wishes to average 60 mph for the entire two-mile trip. Is this possible? (Explain.)



We want

$$60 = \frac{\text{distance}}{\text{time}} = \frac{2 \text{ miles}}{\frac{1}{30} + \frac{1}{x} \text{ hrs}}$$

$$\Rightarrow 60 = \frac{60x}{x+30}$$

$$\Rightarrow 60x = 1800 + 60x, \text{ which has no solution.}$$

Reasoning: For the car to average 60 mph over two miles it needs to travel that distance in $\frac{1}{30}$ hr.