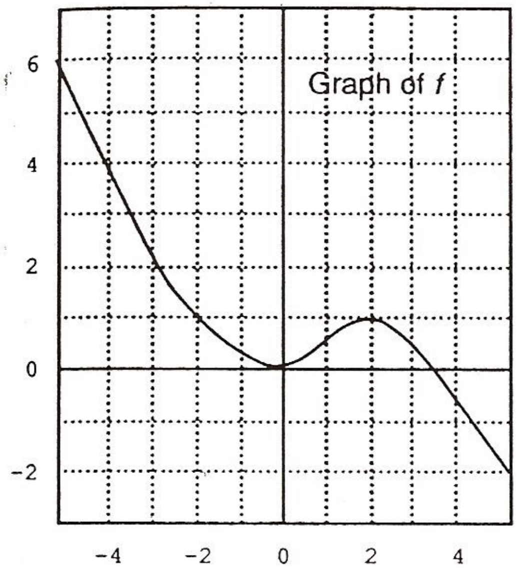
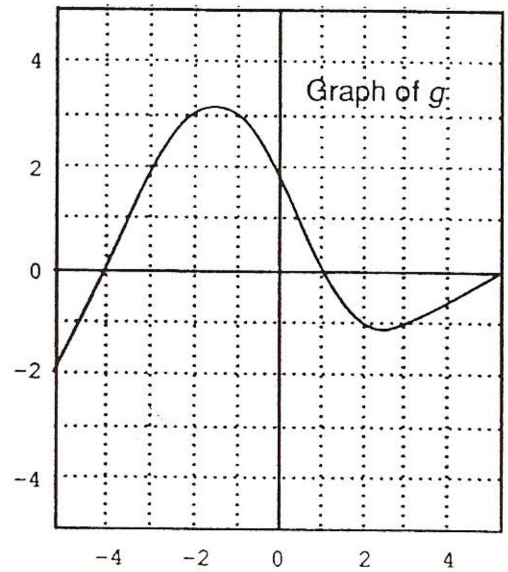
WORKSHEET 6 MATH 1300 February 21, 2008

Goal: To explore the chain rule and its applications.

1. Let $h(x) = f(g(x))$, where f and g have the graphs below.





a. Evaluate $h(-2)$ and $h(3)$.

$h(-2) = f(g(-2)) = f(3) \approx 1/2$

$h(3) = f(g(3)) = f(-1) \approx 1/3$

b. Is $h'(-3)$ positive, negative, or zero? Explain how you know this. (Hint: Use the chain rule to find a formula for $h'(x)$.)

$h'(x) = f'(g(x))\, g'(x)$

$h'(-3) = f'(2)\, g'(-3)$

slope = 0 slope > 0

$h'(-3) = 0$ slope $f'(g(-3)) = 0$

c. Is $h'(1)$ positive, negative, or zero? Explain how you know this.

$h'(1) = f'(g(1))\, g'(1)$

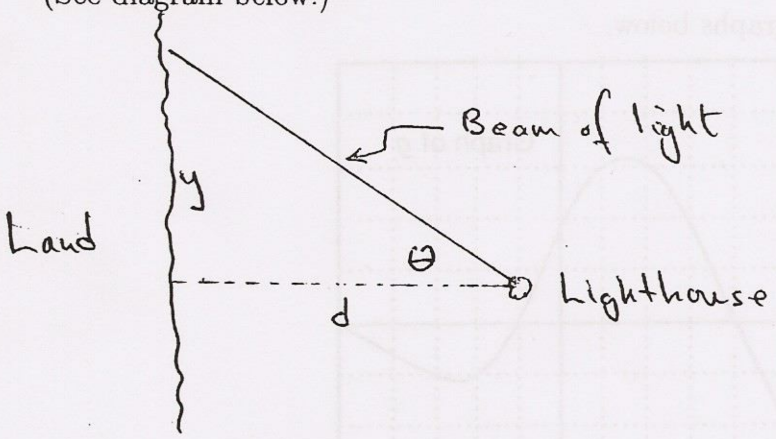
$= f'(0)\, g'(1)$

slope ≥ 0 slope ≤ 0

$h'(1)$ is negative

2. The beacon on a lighthouse, which is located offshore, revolves once every 20 seconds and casts a beam of light on a straight shoreline. When the angle θ between the beam of light and the straight line from the lighthouse to the shore equals 60°, the spot of light on the shore is moving northward at 10 feet per second. Use this information to determine how far the lighthouse is from the shore. (See diagram below.)

$$\frac{d\theta}{dt} = \frac{2\pi}{20s} = \frac{\pi}{10}\ \text{rad/sec}$$



a. Write an equation relating d, y and θ.

$$y = d\tan\theta$$

b. In the above equation, d is a constant but y and θ are functions of time, t. Differentiate both sides of the above equation with respect to the variable t.

$$\frac{dy}{dt} = d\sec^2\theta\,\frac{d\theta}{dt}$$

c. Use your equation in (b) to solve the original problem.

$$10 = d\left(\sec^2\frac{\pi}{3}\right)\left(\frac{\pi}{10}\right)$$

$$\frac{100}{\pi\sec^2\frac{\pi}{3}} = d \qquad \boxed{d = \frac{25}{\pi}\ \text{ft}}$$

d. Suppose the light on the lighthouse is rotating in the opposite direction, so the beam of light is traveling southward along the shore at 10 feet per second. How does this change the problem?

[redacted] $\frac{d\theta}{dt}$ would be negative