WORKSHEET 7 MATH 1300 February 28, 2008

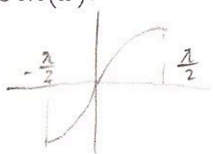
Goal: To find the derivatives of the inverse trig functions.

1. Explain why the function $f(x) = \sin(x)$ does not have an inverse function unless we restrict its domain.

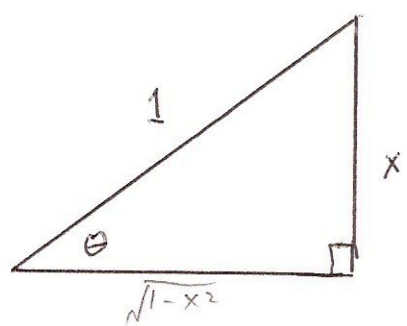
sin(x) is not a 1 - 1 function.

2. Explain why the function $f(x) = \sin(x)$ does have an inverse function if we restrict its domain to the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. (The sine function with this restricted domain is sometimes denoted by $Sin(x)$.

Sin(x) is 1 - 1 function



3. Relationships involving trig and inverse trig functions can sometimes be simplified using elementary *triangle trigonometry.* For example, if we let $\theta = \sin^{-1}(x) = \sin^{-1}(\frac{x}{1})$, with $0 < x < 1$ then we have the triangle:



Use the above triangle to find $\cos(\sin^{-1}(x))$.

$\sin^{-1}(x) = \theta$

$\therefore \cos(\sin^{-1}(x)) = \cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

4. Find $\frac{d}{dx}sin^{-1}(x)$ by differentiating the relationship

$$\sin(\sin^{-1}(x)) = x,$$

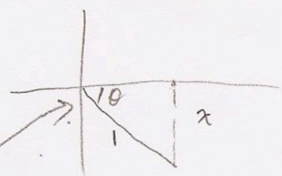
and use the relationship you found in (3) to express $\frac{d}{dx}sin^{-1}(x)$ without using any trig functions.

$$\frac{d}{dx}\sin(\sin^{-1}(x)) = \cos(\sin^{-1}(x)) \cdot \frac{d}{dx}\sin^{-1}(x) = 1$$

$$\Rightarrow \frac{d}{dx}\sin^{-1}(x) = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}}.$$

5. Do we have to change what we did above to find $\frac{d}{dx}sin^{-1}(x)$ for negative values of x, that is for x with $-1 < x < 0$.

No.



If $-1 < x < 0$, $\sin^{-1}(x) \in (-\frac{\pi}{2}, 0)$

We still have $\sin(\sin^{-1}(x)) = x$

Differentiate w.r.t x both sides, we still have $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\cos(\sin^{-1}(x))}$

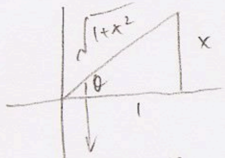
While still: $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$

6. The function $f(x) = \tan(x)$ has an inverse function if we restrict its domain to the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. (Why?) Use the method outlined above to find the derivative of $\tan^{-1}(x)$. (Be careful to describe the domain and range of this function.)

Because it's 1-1 function in $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$\tan(\tan^{-1}x) = x$$

$$\left.\frac{d}{dx}\tan(\tan^{-1}x) = \sec^2(\tan^{-1}x)\cdot\frac{d}{dx}\tan^{-1}(x) = 1\right\} \Rightarrow \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$



$\theta = \tan^{-1}x$

$$\therefore \sec^2(\tan^{-1}x) = \sec^2(\theta) = 1+x^2$$

($\because \theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, it's true for $\theta < 0$, too

by the same argument of Problem 5).