WORKSHEET 8 MATH 1300 March 6, 2008

Goal: To investigate the role of the sign of the first derivative of a function in helping understand its graph

Definition. Let f be a function defined on an interval, and let x_1 and x_2 denote points in that interval.

(a) f is *increasing* on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.
(b) f is *decreasing* on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.
(c) f is *constant* on the interval if $f(x_1) = f(x_2)$ whenever $x_1 < x_2$.

1. Suppose f is a function that is increasing on an interval and x_1 and x_2 are points in the interval. Determine whether the slope of the secant line through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is positive, negative, or zero.

2. In the description of f in problem 1, now assume the function if *decreasing* (resp. *constant*) on an interval, and determine whether the slope of the secant line is positive, negative, or zero.

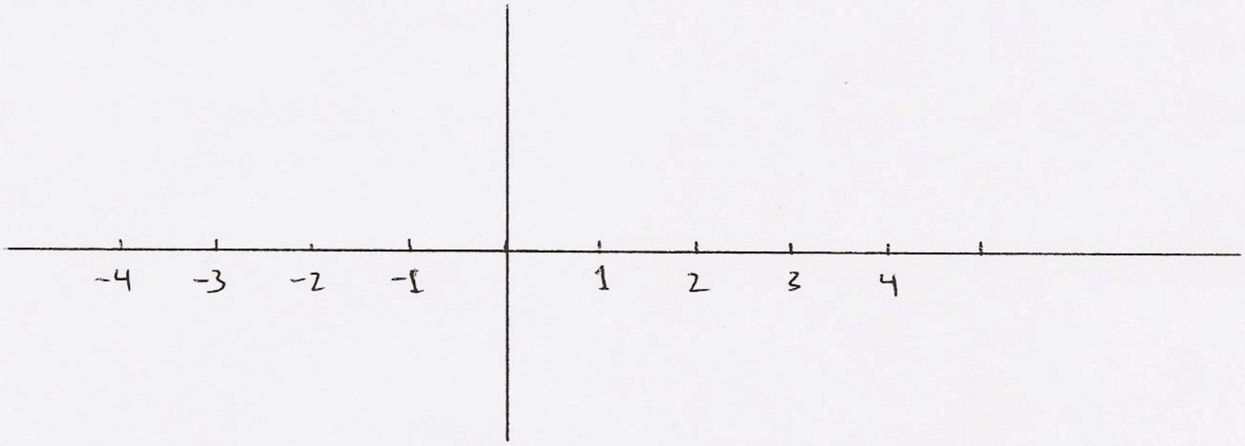
The results discovered for secant lines in problems 1 and 2 carry over to tangent lines. Specifically, we have the following theorem:

Theorem. Let f be a function that is continuous on a closed interval $[a, b]$, and differentiable on the open interval (a, b).

(a) If $f'(x) > 0$ for every value of x in $[a, b]$ then f is *increasing* on $[a, b]$.
(b) If $f'(x) < 0$ for every value of x in $[a, b]$ then f is *decreasing* on $[a, b]$.
(c) If $f'(x) = 0$ for every value of x in $[a, b]$ then f is *constant* on $[a, b]$.

3 Use the above theorem to sketch the graph of a function whose derivative satisfies the properties:

f' is positive on each of the intervals $(-\infty, -1)$ and $(3, \infty)$
f' is zero at $x = -1, x = 1$, and $x = 3$
f' is negative on the intervals $(-1, 1)$ and $(1, 3)$



4 Let $f(x) = |x - 1| + |x + 2|$. Find where f is increasing, where it is decreasing, and where it is constant. Use this information to help sketch the graph of the function