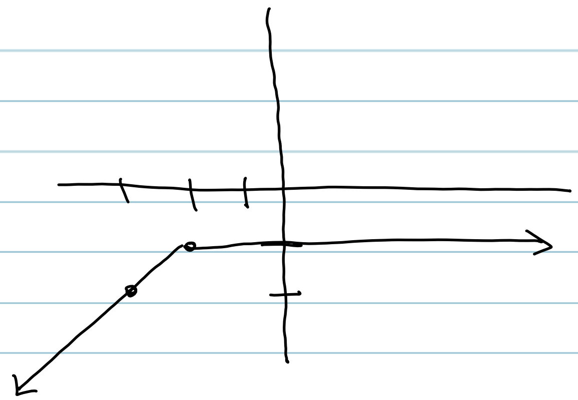
Partial Solutions to Exam 1

1. $y = -f(x+1)$

$-$: reflect over x-axis

$+1$: left one



2. (a) $f(x) = \dfrac{\cancel{(x-a)}(x-b)}{\cancel{(x-a)}(x-c)}$ Graph: $\boxed{A}$

$(x-a)$: hole; $(x-c)$: vert asymptote

(b) $g(x) = \dfrac{\cancel{(x-a)}\cancel{(x-b)}}{\cancel{(x-a)}\cancel{(x-b)}}$ Graph: $\boxed{C}$

$(x-a)$: hole; $(x-b)$: hole

(c) $h(x) = \dfrac{(x-a)(x-b)}{(x-c)(x-d)}$ Graph: $\boxed{B}$

$(x-c)$: vert asymp; $(x-d)$: vert asymp

3. Let $f(x) = 3x^2 - x + 5$ & $g(x) = 2x-1$.
Then

$$
\begin{aligned}
f \circ g(x) &= f(g(x)) \\
&= f(2x-1) \\
&= 3(2x-1)^2 - (2x-1) + 5 \\
&= 3(4x^2 - 4x + 1) - 2x + 1 + 5 \\
&= 12x^2 - 12x + 3 - 2x + 1 + 5 \\
&= \boxed{12x^2 - 14x + 9}
\end{aligned}
$$

4. (a) $\lim_{x \to -2} \dfrac{x+2}{\underbrace{x^2+4}} = \dfrac{-2+2}{(-2)^2+4} = \dfrac{0}{8} = \boxed{0}$

never 0

(graph cont everywhere)

(b) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4} \cdot \dfrac{\sqrt{x} + 2}{\sqrt{x} + 2}$ $\left(\frac{0}{0}\right)$

do more work

$$
\begin{aligned}
&= \lim_{x \to 4} \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)} \\
&= \lim_{x \to 4} \frac{1}{\sqrt{x}+2} \\
&= \frac{1}{\sqrt{4}+2} \\
&= \boxed{\frac{1}{4}}
\end{aligned}
$$

(c) $\lim_{x \to \pi/2} \frac{\cos^2 x}{1 - \sin(x)}$ $\left(\frac{0}{0}\right)$

↑ do more work

$= \lim_{x \to \pi/2} \frac{1 - \sin^2 x}{1 - \sin x}$

$= \lim_{x \to \pi/2} \frac{(1+\sin x)\cancel{(1-\sin x)}}{\cancel{1-\sin x}}$

$= \lim_{x \to \pi/2} 1 + \sin x$

$= 1 + \sin \pi/2$

$= 1 + 1$

$= \boxed{2}$

(d) $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$ $\left(\frac{0}{0}\right)$

↑ do more work

$= \lim_{x \to 0} \frac{1}{x} \cdot \left(\frac{4 - (x+4)}{(x+4)\,4} \right)$

$= \lim_{x \to 0} \frac{1}{x} \cdot \frac{\cancel{4} - x - \cancel{4}}{4(x+4)}$

$= \lim_{x \to 0} \frac{1}{\cancel{x}} \cdot \frac{-\cancel{x}}{4(x+4)}$

$= \lim_{x \to 0} \frac{-1}{4(x+4)}$

$= \cancel{\lim_{x \to 0}} \frac{-1}{4(0+4)} = \boxed{\frac{-1}{16}}$

5. Let

$$f(x) = \begin{cases} \dfrac{-1}{x-2}, & x > -1 \\ x^2+1, & x \le -1 \end{cases}$$

(a) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x^2+1) = \boxed{2}$

(b) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \dfrac{-1}{x-2} = \boxed{\dfrac{1}{3}}$

(c) $\lim_{x \to -1} f(x) = \boxed{\text{DNE}}$ ((a) & (b) do not agree)

(d) $f(-1) = (-1)^2 + 1 = \boxed{2}$

(e) $\lim_{x \to 2^-} f(x) = \boxed{\infty}$

(f) $\lim_{x \to 2^+} f(x) = \boxed{-\infty}$

(g) $\lim_{x \to 2} f(x) = \boxed{\text{DNE}}$ ((e) & (f) do not agree)

(h) $f(2) = \boxed{\text{DNE}}$

(i) $\boxed{\text{discont @ } x = -1, 2}$

6. Recall that

$$-1 \le \cos\left(\frac{2}{x}\right) \le 1.$$

Then

$$-x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4.$$

Notice that

$$\lim_{x \to 0} (-x^4) = 0$$

and

$$\lim_{x \to 0} x^4 = 0.$$

Therefore, by Squeeze Thm,

$$\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = \boxed{0}.$$

7. Pf: Let $\varepsilon > 0$. Choose $\delta = \varepsilon/5$. Assume that

$$0 < |x-1| < \delta.$$

Then

$$\begin{aligned} |f(x) - 7| &= |5x + 2 - 7| \\ &= 5|x-1| \\ &< 5 \cdot \delta \\ &= \cancel{5} \cdot \varepsilon/\cancel{5} \\ &= \varepsilon. \end{aligned}$$

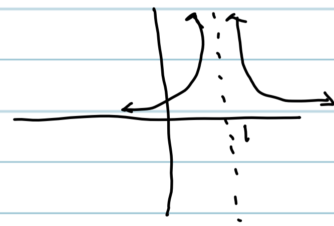
This shows that $|f(x) - 7| < \varepsilon$ whenever $0 < |x-1| < \delta$. ∎

8. <u>Note:</u> There are many correct ans.

(a) $x^2+y^2=1$

$\uparrow$ not a fcn of x

(b) $f(x) = \dfrac{1}{(x-1)^2}$



(c) $h(x) = \dfrac{1}{x-1}$