Lab 1: Exploring limits

Names:

Goal

Develop more intuition about limits by exploring a couple of interesting limits.

Directions

In groups of 2–4, answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). The lab is due on **Monday, Feb 9**.

Exercises

Recall that we say "the limit of $f(x)$ as x approaches a is L" provided that we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side) but not equal to a. We write

$$\lim_{x\to a} f(x) = L.$$

Often, we can gain intuition about a limit by exploring what happens with values "near" $x = a$. As the next two examples will illustrate, sometimes this can be misleading.

1. Let $f(x) = \dfrac{\sqrt{x^2+9}-3}{x^2}$. We would like to find $\lim_{x\to 0} f(x)$. Let's take a look at what happens at some x-values that are pretty close to 0.

   Using your calculator, complete the following table. (Round your answers to 5 decimal places.)

x	$f(x)$
± 1	
± 0.1	
± 0.01	
± 0.001	
± 0.0001	

   From the table, make a guess as to what $\lim_{x\to 0} f(x)$ should be. Write your guess below. (Try to figure out what the actual fraction is, not the decimal.)

It turns out that this really is the right answer. However, now try to see what happens at $x = \pm 0.00000001$ (that's seven zeros and then a 1). What do you get?

Hmmm, what does this mean? Does it mean that our guess above was wrong? (These are rhetorical questions.) The problem is that our calculators just lied to us in the last calculation.

The moral of the story is that just plugging in values is not sufficient for determining limits. Our calculator may lie! We'll come up with techniques for finding limits like this later.

2. Let $g(x) = \sin \frac{\pi}{x}$. Now, we would like to find $\lim_{x \to 0} g(x)$. Let's take a look at what happens at some x-values that are pretty close to 0.

   Using your calculator, complete the following table.

x	$g(x)$
± 0.1	
± 0.01	
± 0.001	
± 0.0001	

   What's your guess for the limit?

   Now, complete this next table using your calculator. (Round your answers to 5 decimal places.)

x	$g(x)$
± 0.3	
± 0.03	
± 0.003	
± 0.0003	

   OK, now what can you conclude?

   There is a picture of the graph of this function on page 69 of our book.