Lab 3: More on u-substitution

Names:

Goal

The goal of this lab is to take a more in-depth look at u-substitution.

Directions

In groups of 2–4, answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). The lab is due on **Wed, May 6** and is worth 10 points.

Quick Review

Let's first remind ourselves what we know so far. The method of u-substitution is a technique for integrating *some* complicated looking functions. By complicated, we mean products, quotients, and compositions of simpler functions. By rewriting the integral in terms of u and du we *hope* that the resulting integral is easier to integrate. It is important to note that u-substitution does not always work.

The steps for u-substitution are:

(i) Choose u. In general, we want to choose u, so that it's derivative will cancel any extra factors involving x. *Usually* (but not always) choose u to be the ____________________________________.

(ii) Find $du = f'(x)dx$ and then find dx.

(iii) Make the appropriate substitutions. You know that you are on the right track if the only variable left is u. If the integral has limits, either keep the old limits (but label them $x = a$ and $x = b$) OR find new limits.

(iv) Do whatever it takes to integrate the new version of the integral in terms of u.

(v) If the integral has no limits, back substitute, so that the answer is in terms of x. If the integral has limits and you kept the old limits, then back substitute and use FTC2. If the integral has limits and you found new limits, then do NOT back substitute; just use the FTC2 in terms of u.

Note that (for now) anytime we are integrating a function that has an "inside" that is more complicated that just x, we will need to make use of u-substitution.

Exercises

Alright, let's try a few more examples.

1. Integrate each of the following indefinite integrals

   (a) $\displaystyle\int \sin(2x)\ dx$

(b) $\int x^2(1-x^3)^{3/2}\ dx$

(c) $\int \sin^2 x \cos x\ dx$

(d) $\int \frac{\sin(1/x)}{x^2}\ dx$

2. Now, let's try a few integrals involving limits.

   (a) Evaluate the following definite integral using u-substitution and keeping the original limits.

$$\int_0^1 \frac{x}{\sqrt{2-x^2}}\,dx$$

   (b) Now, evaluate the same integral, but this time find new limits.

$$\int_0^1 \frac{x}{\sqrt{2-x^2}}\,dx$$

   (c) Evaluate the following definite integral by either keeping the original limits or finding new limits.

$$\int_{-1}^0 \frac{1}{\sqrt{1-3x}}\,dx$$

Occasionally, we can force u-substitution to work when it doesn't look like it should work. Let's work through one such example. Consider the following integral.

$$\int x\sqrt{1+x}\ dx$$

Our guiding principle suggests that we should choose

$$u = \underline{\hspace{3cm}}.$$

In this case, we get

$$du = \underline{\hspace{3cm}},$$

and so

$$dx = \underline{\hspace{3cm}}.$$

What's wrong? Why does it look like u-substitution won't work?

Hmmm, let's see if we can force this to work. If we solve for x in terms of u, we get

$$x = \underline{\hspace{3cm}}.$$

Now, substitute $x+1$, dx, and x with the appropriate quantities and see if you can integrate the function now.

$$\int x\sqrt{1+x}\ dx =$$