MA 2550: Calculus I (Spring 2009)
Review for Exam 1

Exam 1 covers material in sections 2.2–2.5, as well as the material covered in the Diagnostics Tests. This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; you may do what you want with it.

Topics

To be successful on Exam 1 you should

- understand what a function is and be able to determine whether a given graph/equation represents a function (know vertical line test)
- understand what an inverse function is and be able to determine whether a given function has an inverse (know horizontal line test)
- know definitions of the six trigonometric functions, basic trig identities, and be able to evaluate trig functions at the standard angles on the unit circle
- know shapes of basic graphs (including trig functions)
- understand transformations of graphs and be able to sketch transformations of a given graph
- be able to find domain and range
- be able to find combinations of functions (sum, difference, product, quotient, and composition)
- know formulas for slope and equations of lines
- be able to find equation of a line passing through two points
- know intuitive definition of limit (using the word "near")
- be able to come up with examples of graphs AND equations of functions that illustrate various possibilities involving limits (for example, be able to provide an example of an equation such that both the limit from the left and the limit from the right exist, but are not equal)
- know ϵ-δ definition of the limit and be able to use it to prove that a function has a particular limit as x approaches a given value
- know limit laws (including Squeeze Theorem)
- be able to evaluate limits (including problems that require Squeeze Theorem)
- know definition of continuity and have an intuitive understanding of continuity

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.

- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits where they are needed.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.

   (a) The functions $u(x) = \dfrac{\sqrt{x}(x+1)}{x^2+x}$ and $v(x) = \dfrac{1}{\sqrt{x}}$ are equal.

   (b) $f \circ g = g \circ f$ holds for arbitrary functions f and g.

   (c) If a horizontal line does not intersect the graph of a function f, then f does not have an inverse function.

   (d) If a function $f(x)$ does not have a limit as x approaches a from the left, then $f(x)$ does not have a limit as x approaches a from the right.

   (e) If a function is not continuous at $x = c$, then either it is not defined at $x = c$ or it does not have a limit as x approaches c.

   (f) If f is a function such that $f(0) < 0$ and $f(2) > 0$, then there is a number c in the interval $(0, 2)$ such that $f(c) = 0$.

   (g) If $h(x) \leq f(x) \leq g(x)$ for all real numbers x and $\lim_{x\to a} h(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} f(x)$ also exists.

   (h) Exercise 5 in True-False Quiz, page 108.

   (i) Exercise 9 in True-False Quiz, page 108.

(j) Exercise 13 in True-False Quiz, page 108.

2. Determine whether each of the following rules define y as a function of x. If not, provide a reason why.

   (a) $x^2 + 2x + y^3 = 17$

   (b) $x = \sin(y)$

3. Determine whether each of the following scenarios could define a function. If so, identify the domain and range. If not, explain why.

   (a) A set of 10 missiles get launched and each missile follows a set of directions that tell the missile what target to blow up. Some of the missiles land on the same target.

   (b) A person types a phrase into the search engine Google, hits enter, and multiple entries are returned.

4. A camera is mounted at a point on the ground 500 meters from the base of a space shuttle launching pad. The shuttle rises vertically when launched, and the camera's angle of elevation is continually adjusted to follow the bottom of the shuttle. Express the height x as function of the angle of elevation θ.

5. Sketch the graph of each function without using a calculator. In each case, identify how you obtained the graph from the graph of the function in parentheses.

   (a) $f(x) = -1 + \sqrt{2 - x} \quad (y = \sqrt{x})$

   (b) $g(x) = \dfrac{2x - 1}{x} \quad (y = \frac{1}{x}$; Hint: first, split into two fractions.)

6. The graph of $y = a\dfrac{1}{\cos(x + b) + 2} + c$ results when the graph of $y = \dfrac{1}{\cos(x) + 2}$ is reflected over the x-axis, shifted 3 units to the right, and then shifted 4 units down. Find a, b, and c.

7. If $f(x) = 8 - x$ and $g(x) = 3x^2 - x + 4$, find formulas for $(g \circ f)(x)$ and $(f \circ g)(x)$.

8. Sketch the graph of a possible function f that has all properties (a)–(g) listed below.

   (a) The domain of f is $[-1, 2]$

   (b) $f(0) = f(2) = 0$

   (c) $f(-1) = 1$

   (d) $\lim_{x \to 0^-} f(x) = 0$

   (e) $\lim_{x \to 0^+} f(x) = 2$

   (f) $\lim_{x \to 2^-} f(x) = 1$

   (g) $\lim_{x \to -1^+} f(x) = -1$

9. Sketch the graphs of possible functions f, g, and h such that: f satisfies property (a) below, g satisfies property (b) below, and h satisfies property (c) below. There should be three separate graphs.

   (a) $\lim_{x \to 0} f(x) = f(0)$

   (b) $\lim_{x \to 0^-} g(x) = -1$ and $\lim_{x \to 0^+} g(x) = +1$

(c) $\lim_{x\to 0} h(x) \neq h(0)$, where $h(0)$ is defined.

10. Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE).

(a) $\lim_{x\to 2} x^2 + 4x - 12$

(b) $\lim_{x\to 2} \frac{x^2 + 4x - 12}{x^2 + 4x + 3}$

(c) $\lim_{x\to 2} \frac{x^2 + 4x - 12}{x^2 - x - 2}$

(d) $\lim_{x\to 2} \frac{x^2 + 4x - 12}{x^2 - 4x + 4}$

(e) $\lim_{x\to -3} \frac{x}{x+3}$

(f) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

(g) $\lim_{x\to 3} \frac{1}{x-3}$

(h) $\lim_{x\to 3} \frac{1}{(x-3)^2}$

(i) $\lim_{x\to \pi} \frac{x}{\cos x}$

(j) $\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

(k) $\lim_{x\to 5} \frac{|x-5|}{x-5}$

11. Let f be defined as follows.

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ 3x + 4 & \text{if } 0 \leq x \leq 4 \\ x^2 & \text{if } x > 4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{x\to 0^+} f(x)$

(b) $\lim_{x\to 4} f(x)$

12. Let f and g be functions such that $\lim_{x\to a} f(x) = -3$ and $\lim_{x\to a} g(x) = 6$. Evaluate the following limits, if they exist.

(a) $\lim_{x\to a} \frac{(g(x))^2}{f(x) + 5}$

(b) $\lim_{x\to a} \frac{7f(x)}{2f(x) + g(x)}$

(c) $\lim_{x\to a} \sqrt[3]{g(x) + 2}$

13. If $3x \leq f(x) \leq x^3 + 2$ for $0 \leq x \leq 2$, evaluate $\lim_{x\to 1} f(x)$.

14. Use the Squeeze Theorem to prove that $\lim_{x\to 0} x^4 \cos(2/x) = 0$.
15. Prove each of the following using the ϵ-δ definition of limit.
    (a) $\lim_{x\to 1} 2x + 3 = 5$
    (b) $\lim_{x\to 2} x^2 + x = 6$
16. Exercise 40, page 107.
17. Exercise 41, page 107.