MA 2550: Calculus I (Spring 2009)
Review for Exam 3

Exam 3 covers material in sections 4.1–4.5, 4.7, 3.9, 4.9, 5.4, 5.1 and 5.2. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 3 is nearly identical to a question on Exam 2 (so, study Exam 2!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; you may do what you want with it.

Topics

To be successful on Exam 3 you should

- know definition of critical number and be able to find them
- understand the concept of local max/min
- know the relationship between critical numbers and local max/min
- understand the concept of absolute max/min
- know the statement of and understand the Extreme Value Theorem
- be able to find absolute max/min of a continuous function on a closed interval
- know the statement of and understand Rolle's Theorem
- know the statement of and understand the Mean Value Theorem
- be able to find the points guaranteed to exist according to the Mean Value Theorem
- be prepared to provide examples that illustrate various concepts (for example, you should be able to state an example of a function that has a critical number, but does not have a local max or local min)
- understand relationship between the sign of the derivative and increasing/decreasing
- understand relationship between the sign of the second derivative and concavity
- know statement of First Derivative Test and be able to use it to make conclusions about local max/min
- know statement of Second Derivative Test and be able to use it to make conclusions about local max/min; when is this test inconclusive?
- know definition of a point of inflection and be able to find them
- be able to compute limits at infinity; know shortcuts for rational functions
- be able to find vertical, horizontal, and slant asymptotes
- be able to sketch the graph of a function as in the problems we did in Section 4.5; not by copying a picture off your calculator. In reality, I will probably give you a bunch of information about a function and then ask you to sketch the graph from that information. (A great example to look at is the first example I did in class from the supplementary notes from Section 4.5.)
- be able to solve optimization problems

- be able to find differentials
- understand how dy, dx, Δx, and Δy are all related
- be able approximate the change in a function over a particular interval using differentials
- be able to find antiderivatives and evaluate indefinite integrals
- be able to approximate the area (or net signed area) under the graph of a function
- be able to approximate a definite integral of a function over an interval
- know the summation formulas on the top of page 303
- be able to compute the exact value of net signed area under the graph of a function by taking limits of Riemann sums (using right endpoints)
- be able to compute the exact value of a definite integral of a function over an interval by taking limits of Riemann sums (using right endpoints)

Words of advice

Here are somethings to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits where they are needed.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.

   (a) If $f'(c) = 0$, then f has either a local minimum or local maximum at $x = c$.

   (b) If f has a local minimum or local maximum at $x = c$, then $f'(c) = 0$.

   (c) If f is increasing and differentiable on (a, b), then $f'(x) > 0$ for all x in (a, b).

   (d) If f has an inflection point at $x = c$, then $f''(c) = 0$.

   (e) Every rational function has an asymptote that is a polynomial function. (Hint: Don't forget that lines, including horizontal ones are polynomials.)

   (f) If a function f has a vertical asymptote at $x = a$, then the graph of f cannot cross the line $x = a$.

   (g) If a function f has a horizontal asymptote at $y = b$, then the graph of f cannot cross the line $x = b$.

2. Find all critical numbers of $f(x) = x\sqrt{4 - x^2}$.

3. Exercise 11 on page 211

4. Find the absolute max and min for $f(x) = \cos x - x$ on the interval $[0, 2\pi]$.

5. Find the absolute max and min for $g(x) = 2x + \dfrac{1}{2x}$ on the interval $[1, 4]$.

6. Determine whether Rolle's Theorem applies to each of the following functions on the indicated interval. Explain your answer.

   (a) $f(x) = \dfrac{x^3 + x}{x}$, $[-1, 1]$

   (b) $g(x) = \dfrac{x^2}{x^2 - 3}$, $[-3/2, 3/2]$

7. Given $f(x) = 10 - \dfrac{16}{x}$, show that f satisfies the hypotheses of the Mean Value Theorem on the interval $[2, 8]$, and then find all numbers c that the Mean Value Theorem guarantee exist.

8. Exercise 33 on page 220

9. Provide an example of a function f such that $f'(2) = 0$, but f does not have a local maximum or local minimum at $x = 2$.

10. Provide an example of a function f such that $f''(1) = 0$, but f does not have an inflection point at $x = 1$.

11. Assume that a continuous function f has exactly one critical number: $f'(2) = 0$. In each case below, decide whether the point at which $x = 2$ is a local max, local min, or neither.

    (a) $f'(0) = -2$ and $f'(3) = 2$

(b) $f''(2) = -4$

(c) $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$

12. Determine the intervals on which the following functions are increasing, decreasing, concave up, and concave down. Also, find the coordinates of all local maximums, local minimums, and inflection points.

    (a) $f(x) = (x+1)^3(x-2)$

    (b) $g(x) = \cos^2 x$ on interval $(0, \pi)$

13. Evaluate each of the following limits.

    (a) $\lim_{x\to\infty} (-3x^3 + 5x^2 - 3x + 17)$

    (b) $\lim_{x\to\infty} \dfrac{-3x^3 + 5x^2 - 3x + 17}{1 - 2x^3}$

    (c) $\lim_{x\to\infty} \dfrac{-3x^3 + 5x^2 - 3x + 17}{2x^4 + 5x^2 - 4}$

    (d) $\lim_{x\to\infty} \dfrac{-3x^3 + 5x^2 - 3x + 17}{2x^2 + 4x - 6}$

    (e) $\lim_{x\to-\infty} \dfrac{-3x^3 + 5x^2 - 3x + 17}{2x^2 + 4x - 6}$

14. Find *all* asymptotes for each of the following functions.

    (a) $g(x) = \dfrac{x}{x^2 + 4}$

    (b) $h(x) = \dfrac{\sqrt{x^2 + 1}}{x}$

    (c) $f(x) = \dfrac{x^2 + 3x - 2}{x + 2}$

15. Sketch the graph of each of the following functions by finding x-intercepts, y-intercepts, vertical asymptotes, horizontal or slant asymptotes, intervals of increase and decrease, coordinates of turning points (local max/min), intervals of concavity, coordinates of inflection points.

    (a) $f(x) = 8x^3 - 2x^4$

    (b) $g(x) = \dfrac{x^2}{x^2 + 3}$

16. Find two numbers whose sum is 23 and whose product is a maximum.

17. Find the point on the graph of $f(x) = \sqrt{x + 1}$ that is closest to the point $(3, 0)$.

18. Exercise 11, page 263.

19. Exercise 33(a) on page 194

20. Explain the difference between an indefinite integral and a definite integral.

21. Explain why the Power Rule for Integration, $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$, does not apply to the function $f(x) = \dfrac{1}{x}$.

22. Evaluate each of the following indefinite integrals.

    (a) $\int 6\ dx$

    (b) $\int 4x^2 - 5x + 3\ dx$

    (c) $\int 3\sin x\ dx$

    (d) $\int x^5(5 - 2x + 3x^2)\ dx$

    (e) $\int \frac{\cos^3}{1 - \sin^2 x}\ dx$

    (f) $\int \frac{4 + 5x^{3/2}}{\sqrt{x}}\ dx$

    (g) $\int \sec\theta \tan\theta\ d\theta$

23. Find f such that $f'(x) = \sqrt{x}$ and $f(4) = 0$.

24. Approximate the area under the graph of $f(x) = \cos^2 x$ on the interval $[0, \pi]$ by forming 4 rectangles and using right endpoints to determine the height of the rectangles.

25. Evaluate the following definite integrals by taking the limit of Riemann sums. (Use $x_i^* = x_i$; that is, use right endpoints.)

    (a) $\int_0^1 x^2\ dx$

    (b) $\int_{-1}^3 2x - 1\ dx$