Solns to even problems from 8.1

4. $\displaystyle\int x e^{-x}\,dx$ $\qquad u = x \quad v = -e^{-x}$
$\qquad du = dx \quad dv = e^{-x}\,dx$

$= -xe^{-x} - \displaystyle\int (-e^{-x})\,dx$

$= \boxed{-xe^{-x} - e^{-x} + C}$

10. $\displaystyle\int \sin^{-1} x\,dx$ $\qquad u = \sin^{-1} x \quad v = x$
$\qquad du = \dfrac{1}{\sqrt{1-x^2}}\,dx \quad dv = dx$

$= x\sin^{-1}x - \displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ $\qquad u = 1-x^2$
$\qquad du = -2x\,dx$
$\qquad dx = \dfrac{du}{-2x}$

$= x\sin^{-1}x - \displaystyle\int \frac{\cancel{x}}{\sqrt{u}}\,\frac{du}{-2\cancel{x}}$

$= x\sin^{-1}x + \dfrac{1}{2}\displaystyle\int u^{-1/2}\,du$

$= x\sin^{-1}x + \dfrac{1}{2}\cdot 2\,u^{1/2} + C$

$= \boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$

26. $\displaystyle\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$ $\qquad u = \arctan\left(\frac{1}{x}\right) \qquad v = x$

$$du = \frac{-\frac{1}{x^2}}{1+\left(\frac{1}{x}\right)^2}\,dx \qquad dv = dx$$

$$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x \cdot \frac{-\frac{1}{x^2}}{1+\left(\frac{1}{x}\right)^2}\,dx$$

$$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \int_1^{\sqrt{3}} \frac{x}{1+x^2}\,dx \qquad \begin{array}{c} u = 1+x^2 \\ du = 2x\,dx \\ dx = \frac{du}{2x} \end{array}$$

$$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \int_{x=1}^{x=\sqrt{3}} \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \frac{1}{2}\ln|1+x^2|\Big|_1^{\sqrt{3}}$$

$$= \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{2}\ln|4| - \arctan 1 - \frac{1}{2}\ln|2|$$

$$= \boxed{\frac{\pi\sqrt{3}}{3} + \ln 2 - \frac{\pi}{4} - \ln\sqrt{2}}$$

$= $ Some #