Section 12.2: Series (part 2)

In place of having class today, I'd like you to attempt to complete this handout.

First, let's review a little bit.

1. What is a sequence of numbers?

2. What is a series?

3. If $\{a_i\}$ is a sequence, what is the sequence $\{s_n\}$ of partial sums?

4. What does it mean for a series $\sum a_i$ to converge?

5. What is a geometric series? When does a geometric series converge? When a geometric series converges, what does it converge to? When does a geometric series diverge?

6. What is the harmonic series? Does it converge or diverge?

At the end of class yesterday, I said something similar to the following.

Important Note 1. A series may converge or diverge. However, if a series $\sum a_i$ is going to have any chance of converging, the terms of the sequence $\{a_i\}$ must get smaller as i gets larger.

In fact, we have the following theorem. (Try to fill in the blanks. If necessary, look in the textbook for the answer.)

Theorem 2. *If the series $\sum a_i$ converges, then the sequence $\{a_i\}$ converges to* ______________. *That is, if $\sum a_i$ converges,*

$$\lim_{i\to\infty} a_i = \underline{\hspace{3cm}}.$$

It is important to note that the converse of this theorem is *not* true. That is, if a sequence $\{a_i\}$ converges to ____________, the corresponding series $\sum a_i$ does not necessarily converge.

For example, consider the harmonic series. The sequence $\{\frac{1}{i}\}_{i=1}^{\infty}$ converges to ____________, but the series $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i}$ ____________.

However, the contrapositive of the previous theorem is true. (The contrapositive of a conditional statement always has the same truth value. The contrapositive is essentially the result of switching the implication and negating each half.) The contrapositive is the following statement and will be our first line of defense when determining whether a series converges or diverges.

Theorem 3 (Test for Divergence)**.** *If the sequence $\{a_i\}$ does not converge to 0, then the series $\sum a_i$ diverges.*

For example, consider the series $\displaystyle\sum_{i=1}^{\infty} \frac{i}{i+1}$. Since

$$\lim_{i\to\infty} \frac{i}{i+1} = 1 \neq 0,$$

the corresponding series must diverge.

If you want to know whether a series converges or diverges, the first thing you should check is whether the corresponding sequence converges to 0. If it doesn't, then the series diverges and you are done. However, if the sequence does converge to 0, we'll need to do more work.

See you next week!