MA 2560: Calculus II (Spring 2009)
Review for Exam 1

Exam 1 covers material from sections 7.1, 7.2*–7.4*, 7.6–7.8, and 8.1. This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; do what you want with it.

Topics

To be successful on Exam 1 you should

- know definition of a one-to-one function and be able to determine whether a given function is one-to-one.
- know how to apply the horizontal line test and be able to use it to make conclusions about a given function.
- know definition of inverse function (i.e., If f has an inverse, then $f^{-1}(y) = x$ iff $f(x) = y$).
- know how to determine whether a given function has an inverse. (Be prepared to generate examples of functions that have inverses and examples of functions that do not have inverses.)
- know how to find the graph of an inverse function (if it exists) and how to find the corresponding equation.
- understand relationship between domain and range for two functions that are inverses of each other.
- be able to find the derivative at a point for the inverse of a given function even if you cannot determine the inverse function (see Theorem 7 on page 389).
- know definition of the natural log function as an integral.
- know the formula for the derivative of the natural log and be able to apply it in conjunction with the chain rule.
- know the basic properties of logs.
- know the definition of e.
- know formula 8 on page 426 and be able to apply it in conjunction with u-substitution.
- know how to take derivatives using the technique called logarithmic differentiation.
- know the definition of the natural exponential function.
- understand the relationship between the natural log function and the natural exponential function. ("The output of a log is an exponent.")
- know the formula for the derivative (respectively, integral) of the natural exponential function in conjunction with the chain rule (respectively, u-substitution).
- know the basic shapes of the natural log function and the natural exponential function. (How are these two graphs related?)
- know the definitions of arcsin, arccos, and arctan.

- be able to apply the formulas for the derivatives of the inverse trig functions and the corresponding integral formulas. (I will provide the necessary formulas).
- know the definitions of sinh, cosh, and tanh.
- know the identity $\cosh^2 x - \sinh^2 x = 1$.
- be able to apply the formulas for the derivatives of the hyperbolic functions and the corresponding integral formulas. (I will provide the necessary formulas).
- be able to apply the formulas for the derivatives of the inverse hyperbolic functions and the corresponding integral formulas. (I will provide the necessary formulas).
- be able to recognize various indeterminate forms when dealing with limits.
- know statement of L'Hospital's Rule and be able to apply it in various types of examples.
- know the formula for integration by parts and be able to integrate functions using integration by parts by making appropriate choices for u and dv.

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits, integral symbols, $+C$, etc. where they are needed.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. You do not necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
   (a) If f is a one-to-one function with domain consisting of all real numbers, then $f^{-1}(f(17)) = 17$.
   (b) If a differentiable function f has at least one local maximum, then f does not have an inverse.
   (c) $\sin^{-1} x = \sec x$.
   (d) For all real numbers x, $\arcsin(\sin x) = x$.
   (e) The derivative of a rational function $r(x) = \dfrac{f(x)}{g(x)}$ (where $f(x)$ and $g(x)$ are polynomials) is a rational function.
   (f) The (indefinite) integral of a rational function $r(x) = \dfrac{f(x)}{g(x)}$ (where $f(x)$ and $g(x)$ are polynomials) is a rational function.
   (g) If f and g are differentiable on an open interval containing a, except possibly at $x = a$, and $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ is an indeterminate form of type ∞/∞, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$.
2. If f is a one-to-one function such that $f(-2) = 3$, what is $f^{-1}(3)$? Can you make the same conclusion if f is not one-to-one?
3. Exercise 19, page 391.
4. Exercise 20, page 391.
5. Find the inverse function of $f(x) = 2 + \ln\left(1 - \frac{1}{x}\right)$.
6. If $f(x) = 2x + \sin x$, find $(f^{-1})'(\pi)$.
7. Exercise 41, page 392.
8. Exercise 11, page 435.
9. Solve for x: $\ln(x + 1) + \ln(x - 1) = 1$.
10. Differentiate each of the following functions.
    (a) $y = \ln x^2$
    (b) $f(x) = (\ln x)^2$
    (c) $g(x) = x^e$
    (d) $y = x^x$
    (e) $y = \ln(\cos x)$
    (f) $f(x) = x^2 e^{3x+5}$
    (g) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ (What did you just prove?)

(h) $f(x) = \tan^{-1}\sqrt{x}$
(i) $g(x) = \sqrt{\tan^{-1} x}$
(j) $y = \arccos(e^x)$
(k) $h(x) = \sinh\left(1 + e^{-3x}\right)$

11. Use logarithmic differentiation to find the derivative of each of the following functions.

(a) $f(x) = \dfrac{(3-x)^{1/3}\, x^2}{(1-x)(3+x)^{2/3}}$.
(b) $y = (x+1)(e^{x^2} + 1)$

12. Let $f(x) = \arcsin(x)$. Find the equation of the tangent line to f at $x = \dfrac{1}{2}$.

13. Let $g(x) = x \ln x$. Find the equation of the tangent line to g at the point (e, e).

14. Exercise 49, page 462.

15. Evaluate each of the following limits:

(a) $\displaystyle\lim_{x\to(\pi/2)^+} e^{\tan x}$
(b) $\displaystyle\lim_{x\to 0^+} \arctan\left(\frac{1}{x}\right)$
(c) $\displaystyle\lim_{x\to\pi/2} \frac{1-\sin(x)}{\cos(x)}$
(d) $\displaystyle\lim_{x\to\infty} \frac{x^3 - x}{e^x}$
(e) $\displaystyle\lim_{x\to 0^+} \frac{\sin x}{\ln x}$
(f) $\displaystyle\lim_{x\to 0^+} x \ln x$
(g) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$
(h) $\displaystyle\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$

16. Exercise 85, page 480.

17. Let $f(x) = |x|$ and $g(x) = \sin x$. Can L'Hôpital's Rule be used to evaluate $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)}$? If so, what is the limit? If not, justify your answer.

18. Integrate each of the following indefinite or definite integrals. For the definite integrals, you should give *exact* answers (i.e., not decimal approximations using your calculator).

(a) Exercise 67, page 429.
(b) $\displaystyle\int \frac{(\ln x)^3}{x}\, dx$
(c) $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$
(d) $\displaystyle\int \tan x\, dx$ (Hint: rewrite $\tan x$ using an appropriate trig identity, then use ordinary u-substitution.)

(e) $\displaystyle\int_0^{\pi} \sin(x)e^{\cos(x)}\ dx$

(f) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\ dx$

(g) $\displaystyle\int \frac{x}{9x^2+4}\ dx$

(h) $\displaystyle\int \frac{1}{9x^2+4}\ dx$

(i) $\displaystyle\int \frac{9x^2+4}{x}\ dx$

(j) $\displaystyle\int \frac{x}{\sqrt{9x^2+4}}\ dx$

(k) $\displaystyle\int \frac{1}{\sqrt{9x^2-4}}\ dx$

(l) $\displaystyle\int \frac{1}{x\sqrt{9x^2-4}}\ dx$

(m) $\displaystyle\int \frac{1}{\sqrt{4-9x^2}}\ dx$

(n) $\displaystyle\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\ dx$

(o) $\displaystyle\int \frac{1-x^2}{\arcsin x}\ dx$

(p) $\displaystyle\int e^x \sin x\ dx$

(q) $\displaystyle\int_0^{2\pi} x\cos x\ dx$

(r) $\displaystyle\int \ln x\ dx$

(s) $\displaystyle\int x^2 \sin x\ dx$

(t) $\displaystyle\int \sec^3 x\ dx$ (Hint: rewrite $\sec^3 x$ as $\sec^2 x \sec x$ and use integration by parts.)

19. Exercise 63, page 495.