MA 2560: Calculus II (Spring 2009)
Review for Exam 2

Note: Anything in red has been added since the original posting of the review.

Exam 1 covers material from sections 8.2–8.5, 8.7, 8.8, 9.1, and 9.2. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; do what you want with it.

Topics

To be successful on Exam 2 you should

- be able to evaluate a variety of integrals using the techniques that we have covered so far this semester. In particular, you should be able to:
  - be able to evaluate trigonometric integrals (i.e., powers of sine times powers of cosine, powers of secant times powers of tangent, or a mixture of all 4)
  - be able to evaluate integrals that require trigonometric substitution
  - be able to evaluate integrals of rational functions by the method of partial fractions (don't forget to check to see if you need to do polynomial long division)
  - be able to evaluate integrals using the integral formulas that we have discussed (I will provide the same list of formulas that I provided on the previous exam)
- be able to approximate definite integrals of continuous function using Midpoint Rule, Trapezoid Rule, and Simpson's Rule (I will provide the necessary formulas)
- be able to determine whether an improper integral converges or diverges
- be able to determine the value of improper integrals that converge (use of proper notation is important)
- be able to find the arc length of a given curve over a specified interval (you should memorize the appropriate formula)
- be able to find the surface area of a solid of revolution obtained by revolving a given curve about the x-axis or y-axis (you should memorize the appropriate formula)

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.

- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits, integral symbols, $+C$, etc. where they are needed.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. You do not necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.

   (a) If f is continuous, then $\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\,dx$.

   (b) Every elementary function has an elementary derivative.

   (c) Every elementary function has an elementary antiderivative.

   (d) If f is continuous on $[0,1]$ with $f''(x) > 0$ on $[0,1]$, then the approximation of $\int_0^1 f(x)\,dx$ by the Trapezoid Rule will be an overestimation.

   (e) If f is continuous on $[0,\infty)$ and $\int_1^{\infty} f(x)\,dx$ converges, then $\int_0^{\infty} f(x)\,dx$ converges.

   (f) If f is a continuous and decreasing function on $[1,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^{\infty} f(x)\,dx$ converges.

   (g) If $f(x) \leq g(x)$ and $\int_0^{\infty} g(x)\,dx$ diverges, then $\int_0^{\infty} f(x)\,dx$ also diverges.

   (h) If $\int_0^1 f(x)\,dx$ converges, then the arc length of $y = f(x)$ on $[0,1]$ is finite.

2. Integrate each of the following functions using an appropriate technique.

   (a) $\displaystyle\int \frac{\cos^5 x}{\sin x}\,dx$

(b) $\int \cos^2 x \sin^2 x \, dx$

(c) $\int \frac{\sin^3 x}{\sqrt{\cos x}} \, dx$

(d) $\int \sec^4 x \tan^3 x \, dx$

(e) $\int \sec^2 x \tan^2 x \, dx$

(f) $\int \frac{4x^2 - 9}{x} \, dx$

(g) $\int \frac{\sqrt{4x^2 - 9}}{x} \, dx$

(h) $\int \frac{x}{\sqrt{4x^2 - 9}} \, dx$

(i) $\int \frac{8x^3}{4x^2 - 9} \, dx$

(j) $\int \frac{1}{\sqrt{4x^2 + 9}} \, dx$

(k) $\int \frac{4x^2}{\sqrt{9 - 4x^2}} \, dx$

(l) $\int \frac{1}{4x^2 + 9} \, dx$

(m) $\int \frac{1}{(x^2 + 4)^{3/2}} \, dx$

(n) $\int \frac{x^3 - x + 3}{x^2 + x - 2} \, dx$

(o) $\int \frac{9x^3 + 12x}{(x^2 + 1)^2} \, dx$

(p) $\int \frac{x + 1}{x^2 + 2x + 5} \, dx$ (Hint: this one isn't that hard)

(q) $\int \frac{x^3 + 5x^2 + 6x - 2}{x^2 + 3x - 4} \, dx$

(r) $\int \frac{2x^2 + x + 8}{x^4 + 8x^2 + 16} \, dx$

3. For each of the following improper integrals, state why the integral is improper and then determine whether the integral converges or diverges. If the integral converges, determine its value.

(a) $\int_0^\infty \frac{1}{4 + x^2} \, dx$

(b) $\int_0^\infty e^{-x} \, dx$

(c) $\int_{-1}^1 \frac{1}{x^2} \, dx$

(d) $\int_0^1 \frac{1}{\sqrt{x}} \, dx$

(e) $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx$

4. If f' is continuous on $[0, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, show that $\int_0^{\infty} f'(x)\, dx = -f(0)$.

5. Find the arc length of the given curve on the specified interval.

   (a) $f(x) = \frac{2}{3}x^{3/2} + 4$ on $[0, 1]$

   (b) $y = \frac{1}{3}(x^2 + 2)^{3/2}$ on $[0, 3]$

   (c) Exercise 13, page 566

6. Using the formula for arc length, prove that the circumference of the unit circle is 2π. (Hint: find the arc length of the quarter of the circle in the first quadrant and multiply by 4; the resulting integral is improper, so don't forget to take that into account.)

7. Exercise 63, page 555 (use $n = 6$ instead of $n = 10$)

8. Exercise 64, page 555 (use $n = 6$ instead of $n = 10$)

9. Exercise 67, page 556

10. Exercise 68, page 556

11. Exercise 71, page 556

12. Exercise 5, page 573

13. Exercise 10, page 573

14. Exercise 15, page 573