Additional example for Section 7.6

Here is another integral example that illustrates the kinds of things that you may encounter.

Example 1. Evaluate the integral: $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx$.

At first glance, this looks like the formula that equals $\arcsin(u)+C$. However, the 4 is causing problems. Here is one way of handling this situation. (Note: I will show you another and easier way later.)

$$\begin{aligned}
\int \frac{1}{\sqrt{4-x^2}}\,dx &= \int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx \\
&= \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx \\
&= \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,2du && u=\frac{x}{2}, du=\frac{dx}{2}, dx=2du \\
&= \int \frac{1}{\sqrt{1-u^2}}\,du \\
&= \arcsin(u)+C \\
&= \arcsin\left(\frac{x}{2}\right)+C
\end{aligned}$$

This problem is very similar to Exercise 7.6.66. On that problem, you will have to make use of the same "factoring trick," but you'll try to make the integral look like the one that equals $\operatorname{arcsec}(u)+C$.

This work is licensed under the Creative Commons Attribution-Share Alike 3.0 License. Written by D.C. Ernst