Section 8.3: Trigonometric Substitution

Goal

In this section, we will introduce a technique of integration called *trigonometric substitution*. This technique is useful for dealing with functions containing the forms: $a^2 - u^2$, $a^2 + u^2$, and $u^2 - a^2$.

The 3 basic trig substitutions

form	substitution	identity	triangle
$a^2 - u^2$	$u = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$	
$a^2 + u^2$	$u = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$	
$u^2 - a^2$	$u = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$	

Important Note 1.

1. The pattern of constant^2, variable^2, and sign matches each identity.
2. You should always try u-sub before attempting trig sub. Also, sometimes you may already know a formula for evaluating the integral.

Examples

OK, let's jump in and do a bunch of examples.

Example 2. Integrate.

(a) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

 Written by D.C. Ernst

(b) $\displaystyle\int \frac{1}{(4x^2+9)^2}\, dx$

 Written by D.C. Ernst

(c) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx$

(d) $\displaystyle\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

(e) $\displaystyle\int \frac{x}{\sqrt{x^2-25}}\,dx$

(f) $\displaystyle\int \frac{1}{\sqrt{x^2-25}}\,dx$