MA 2560: Calculus II (Spring 2010)
Review for Exam 2

Exam 2 covers material from sections 7.8, 8.1–8.5. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; do what you want with it.

You will be provided with the same formulas on this exam as you were on the previous exam.

Topics

To be successful on Exam 2 you should

- be able to recognize various indeterminate forms when dealing with limits.
- know statement of L'Hospital's Rule and be able to apply it in various types of examples.
- be able to evaluate a variety of integrals using the techniques that we have covered so far this semester. In particular, you should be able to:
  - be able to evaluate integrals using integration by parts by making appropriate choices for u and dv.
  - be able to evaluate trigonometric integrals (i.e., powers of sine times powers of cosine, powers of secant times powers of tangent, or a mixture of all 4)
  - be able to evaluate integrals that require trigonometric substitution
  - be able to evaluate integrals of rational functions by the method of partial fractions (don't forget to check to see if you need to do polynomial long division)
  - be able to evaluate integrals using the integral formulas that we have discussed (I will provide the same list of formulas that I provided on the previous exam)

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points. On the other hand, if you have the wrong answer because of a silly computational mistake, but have shown that you have an understanding of the material being tested, then you will receive most of the points.
- I will be grading the justification of your answer, not just the answer. So, you must use proper notation and make appropriate conclusions.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.

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- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits, integral symbols, $+C$, etc. where they are needed. This goes along with using proper notation and making appropriate conclusions.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. You do not necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that this concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
   (a) The derivative of a rational function $r(x) = \dfrac{f(x)}{g(x)}$ (where $f(x)$ and $g(x)$ are polynomials) is a rational function.
   (b) The (indefinite) integral of a rational function $r(x) = \dfrac{f(x)}{g(x)}$ (where $f(x)$ and $g(x)$ are polynomials) is a rational function.
   (c) If f and g are differentiable on an open interval containing a, except possibly at $x = a$, and $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ is an indeterminate form of type ∞/∞, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$.
   (d) Every elementary function has an elementary derivative.
   (e) Every elementary function has an elementary antiderivative.
2. Evaluate each of the following limits:
   (a) $\lim_{x\to 0^+} \arctan\left(\dfrac{1}{x}\right)$
   (b) $\lim_{x\to \pi/2} \dfrac{1-\sin(x)}{\cos(x)}$
   (c) $\lim_{x\to \infty} \dfrac{x^3 - x}{e^x}$

(d) $\lim_{x\to 0^+} \dfrac{\sin x}{\ln x}$

(e) $\lim_{x\to 0^+} x \ln x$

(f) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$

(g) $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$

3. Exercise 85, page 480.

4. Let $f(x) = |x|$ and $g(x) = \sin x$. Can L'Hôpital's Rule be used to evaluate $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$? If so, what is the limit? If not, justify your answer.

5. Integrate each of the following functions using an appropriate technique.

(a) $\displaystyle\int \frac{(\ln x)^3}{x}\, dx$

(b) $\displaystyle\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\, dx$

(c) $\displaystyle\int x^2 \sin x\, dx$

(d) $\displaystyle\int e^x \sin x\, dx$

(e) $\displaystyle\int_0^{2\pi} x \cos x\, dx$

(f) $\displaystyle\int \ln x\, dx$

(g) $\displaystyle\int \frac{x+2}{x^2+9}\, dx$

(h) $\displaystyle\int \frac{\cos^5 x}{\sin x}\, dx$

(i) $\displaystyle\int \cos^2 x \sin^2 x\, dx$

(j) $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\, dx$

(k) $\displaystyle\int \sec^3 x\, dx$ (Hint: rewrite $\sec^3 x$ as $\sec^2 x \sec x$ and use integration by parts.)

(l) $\displaystyle\int \sec^4 x \tan^3 x\, dx$

(m) $\displaystyle\int \sec^2 x \tan^2 x\, dx$

(n) $\displaystyle\int \frac{4x^2-9}{x}\, dx$

(o) $\displaystyle\int \frac{\sqrt{4x^2-9}}{x}\, dx$

(p) $\displaystyle\int \frac{x}{\sqrt{4x^2-9}}\,dx$

(q) $\displaystyle\int \frac{8x^3}{4x^2-9}\,dx$

(r) $\displaystyle\int \frac{1}{\sqrt{4x^2+9}}\,dx$

(s) $\displaystyle\int \frac{4x^2}{\sqrt{9-4x^2}}\,dx$

(t) $\displaystyle\int \frac{1}{4x^2+9}\,dx$

(u) $\displaystyle\int \frac{1}{(x^2+4)^{3/2}}\,dx$

(v) $\displaystyle\int \frac{x^3-4x-10}{x^2-x-6}\,dx$

(w) $\displaystyle\int \frac{x^2-2x-1}{x^3-x}\,dx$

(x) $\displaystyle\int \frac{x+1}{x^2+2x+5}\,dx$

(y) $\displaystyle\int \frac{x^2-x+6}{x^3+3x}\,dx$

(z) $\displaystyle\int \frac{x}{x^2+2x+1}\,dx$