MA 2560: Calculus II (Spring 2010)
Review for Exam 3

Exam 3 covers material from sections 8.7, 8.8, 9.1, 9.2, 11.1, 11.2, 11.3, 11.4, ~~and 12.1~~. Material covered on the previous exam is also fair game. This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions given here. I will not collect this review; do what you want with it.

You will be provided with the same formulas on this exam as you were on the previous exam.

Topics

To be successful on Exam 3 you should

- be able to determine whether an improper integral converges or diverges
- be able to determine the value of improper integrals that converge (use of proper notation is important)
- be able to approximate definite integrals of continuous function using Midpoint Rule, Trapezoid Rule, and Simpson's Rule (I will provide the necessary formulas)
- be able to find the arc length of a given curve over a specified interval (you should memorize the appropriate formula)
- be able to find the surface area of a solid of revolution obtained by revolving a given curve about the x-axis or y-axis (you should memorize the appropriate formula)
- understand parametric equations
- be able to find the direction of a parametric curve
- be able to convert from parametric form to rectangular form by eliminating the parameter
- be able to find dy/dx for a pair of parametric equations (in particular, you should be able to find horizontal and vertical tangents)
- be able to find the equation of tangent line(s) to a parametric curve for a given value of t
- be able to find area, arc length, and surface area for problems involving parametric curves (you should know how to do the appropriate substitutions, e.g., $\sqrt{1+(dy/dx)^2}\ dx = \sqrt{(dx/dt)^2+(dy/dt)^2}\ dt$)
- understand the polar coordinate system
- be able to convert back and forth from polar form to rectangular form (in particular, you should know: $x = r\cos\theta, y = r\sin\theta, x^2+y^2=r^2, \tan\theta = y/x$)
- be able to sketch the graph of a polar equation by hand and with your graphing calculator (in particular, you should be able to determine a smallest interval of θ that traces out the graph of a polar equation once)
- be able find points of intersection of two polar graphs
- be able to find area for problems involving polar equations
- ~~know the definition of a sequence~~
- ~~understand the notation of sequences~~

- ~~be able to find the limit of a sequence~~
- ~~understand what increasing, decreasing, monotone, and bounded sequences are and be able to determine whether a given sequence has one of these properties~~
- ~~know statement of and understand the Monotonic Sequence Theorem~~

Words of advice

Here are a few things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show how and why you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points. On the other hand, if you have the wrong answer because of a silly computational mistake, but have shown that you have an understanding of the material being tested, then you will receive most of the points.
- I will be grading the justification of your answer, not just the answer. So, you must use proper notation and make appropriate conclusions.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits, integral symbols, $+C$, etc. where they are needed. This goes along with using proper notation and making appropriate conclusions.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. You do not necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that this concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
   (a) If f is continuous on $[0,1]$ with $f''(x) > 0$ on $[0,1]$, then the approximation of $\int_0^1 f(x)\,dx$ by the Trapezoid Rule will be an overestimation.
   (b) If f is continuous, then $\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_{-t}^{t} f(x)\,dx$.
   (c) If f is continuous on $[0,\infty)$ and $\int_1^{\infty} f(x)\,dx$ converges, then $\int_0^{\infty} f(x)\,dx$ converges.
   (d) If f is a continuous and decreasing function on $[1,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^{\infty} f(x)\,dx$ converges.
   (e) If $f(x) \leq g(x)$ and $\int_0^{\infty} g(x)\,dx$ diverges, then $\int_0^{\infty} f(x)\,dx$ also diverges.
   (f) If $\int_0^1 f(x)\,dx$ converges, then the arc length of $y = f(x)$ on $[0,1]$ is finite.
2. Exercise 15, page 541
3. Exercise 35, page 543
4. For each of the following improper integrals, state why the integral is improper and then determine whether the integral converges or diverges. If the integral converges, determine its value.
   (a) $\int_0^{\infty} \frac{1}{4+x^2}\,dx$
   (b) $\int_0^{\infty} e^{-x}\,dx$
   (c) $\int_{-1}^{1} \frac{1}{x^2}\,dx$
   (d) $\int_0^{1} \frac{1}{\sqrt{x}}\,dx$
   (e) $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx$
5. If f' is continuous on $[0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, show that $\int_0^{\infty} f'(x)\,dx = -f(0)$.
6. Find the arc length of the given curve on the specified interval.
   (a) $f(x) = \frac{2}{3}x^{3/2} + 4$ on $[0,1]$
   (b) $y = \frac{1}{3}(x^2+2)^{3/2}$ on $[0,3]$
   (c) Exercise 13, page 566
7. Using the formula for arc length, prove that the circumference of the unit circle is 2π. (Hint: find the arc length of the quarter of the circle in the first quadrant and multiply by 4; the resulting integral is improper, so don't forget to take that into account.)
8. Exercise 5, page 573

9. Exercise 10, page 573
10. Exercise 13, page 573
11. Exercise 7, page 662
12. Exercise 11, page 662
13. Exercise 37, page 664
14. Exercise 17, page 672
15. Exercise 25, page 672
16. Exercise 33, page 673
17. Exercise 41, page 673
18. Exercise 59, page 674
19. Exercise 17, page 684
20. Exercise 19, page 684
21. Exercise 23, page 684
22. Exercise 49, page 684
23. Exercise 37, page 689
24. Exercise 17, page 689
25. Exercise 23, page 689
26. ~~Exercise 1, page 730~~
27. ~~Exercise 5, page 720~~
28. ~~Exercise 13, page 720~~
29. ~~Exercise 19, page 721~~
30. ~~Exercise 29, page 721~~
31. ~~Exercise 42, page 721~~